For this tutorial, you will need the MATLAB m-files Unlimited.m and Logistic.m, available on our MATLAB webpage. You will need to download and save these m-files before you can use them.

1. Constant per capita (also known as unlimited or exponential) growth

This question considers the unlimited growth initial value problem:

$$R' = 0.1R; \qquad R(0) = 2{,}000$$

(R is in rabbits; R' is in rabbits per month).

Run the m-file Unlimited.m (note: in order to run Unlimited.m you will need to specify $tmax$ (number of months), N (number of time steps), k (per capita growth rate), and $R(0)$ (initial population)). To do this, in the Command Window we enter:

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Unlimited(30,60,.1,2000)
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in the Command Window. This will produce, on the same set of axes:

(i) A graph (in green circles) of an approximate solution (obtained using Euler's method) to the above initial value problem; and

(ii) A graph (in blue) of the exact, closed form solution to the above initial value problem.

Answer these questions:

(a) Write down the formula for the exact solution to the above IVP. (Use what you know about the unlimited growth IVP, and/or refer to the MATLAB code.)

(b) What *single parameter* that you pass to your program Unlimited.m would you change, to make the numerical and closed-form solutions agree more closely? Go ahead and make that change, and run it again to make sure things worked. Continue to adjust as necessary until the two solutions appear to fit each other as closely as possible.

(c) What happens to a population of 2,000 rabbits after 6 months, and after 2 years? (Use your exact solution; check your answers against what you see on the graph.)

(d) How long does it take the rabbit population to reach 25,000? (Again, use your exact solution, and check your answer against what you see on the graph.)

2. Logistic growth

The following questions concern a rabbit population described by the logistic model

$$R' = 0.1R\left(1 - \frac{R}{25{,}000}\right)$$

rabbits per month. For this exercise, you will need to use the program Logistic.m.

(Your program Logistic.m should retain the same stepsize that you ended up with in exercise 1(b) above.) Run the program Logistic.m (remember to pass the appropriate parameters to the m-file). You should get a graph that looks like the logistic growth curves we have discussed in class.

Answer these questions:

(a) Under this logistic growth model, what happens to a population of 2,000 rabbits after 6 months, after 2 years, and after 5 years? Read these values off the graph as well as you can.

(b) Run your program Logistic.m again, but suppose that your starting number of rabbits is now 40,000 instead of 2,000. You will want to change line (18) (the last line) of Logistic.m so that the y-axis now goes from $b - 1000$ to 45,000. Compare your new graph to the previous logistic graph. How do the graphs differ? In what ways are they similar?

(c) Fill in the blanks: In a logistic growth situation, the population will increase if the initial population is ___________ than the carrying capacity, and will decrease if the initial population is _____________ than the carrying capacity. This makes sense because the _______________________ tells us how large a population the environment can support. It also makes mathematical sense because, if the initial population is smaller than the carrying capacity, then the quantity R/b will initially be ___________ than 1, so the derivative

$$R' = R\left(1 - \frac{R}{b}\right)$$

will initially be positive, whereas, if the initial population is _____________ than the carrying capacity, then the quantity R/b will initially be greater than 1, so R' will initially be _______________.