

M403K Third Midterm Exam
April 11, 2002

1. Related rates.

Consider the curve $y^2 = x^3 + 1$, $y \geq 0$.

- a) Find the slope of the line that is tangent to the curve at the point $(2,3)$.
- b) A particle is moving along the curve. Its x -coordinate is increasing at a rate of 10 units/second. How fast is y changing when $(x, y) = (2, 3)$?

Problem 2. L'Hopital's Rule Evaluate the following limits:

- a) $\lim_{x \rightarrow \infty} \frac{15x^2 - 9}{x^3 + 3x^2 + 5}$
- b) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$
- c) $\lim_{x \rightarrow 2} \frac{\ln(x) - \ln(2)}{x - 2}$
- d) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x^2 + 1}$

Problem 3. Elasticity of Demand

The demand x for a new toy depends on its price p via the demand equation

$$x = 1000e^{-p}.$$

- a) Compute the elasticity of demand $E(p)$ as a function of p .
- b) For what values of p is the demand elastic? For what values of p is the demand inelastic?
- c) What value of p will maximize revenue?

Problem 4. Horse sense

For the first two years of life, a pony's height $H(t)$ grows at a rate

$$H'(t) = 15 - 3t^2,$$

(where height is measured in inches and time in years). At age 1, the pony is 45 inches tall.

- a) How tall was the pony at birth?

b) How tall will the pony be at age 2?

Problem 5. Indefinite integrals.

Evaluate the following integrals:

a) $\int (2x + e^x) dx$

b) $\int xe^{x^2} dx$

c) $\int \frac{\ln(x)}{x} dx$

d) $\int (2x + 1)^4 dx$.

Problem 6. Area under a curve.

We are interested (OK, OK, your instructor is interested) in finding the area under the curve $y = 2x^2 + 1$ between $x = 1$ and $x = 4$.

a) Estimate this area using 3 rectangles. Your final answer should be an explicit number, like 13 or 152.

b) Estimate the area using N rectangles. You can leave your answer as a sum, like $\sum_{k=1}^N 4(\ln(N) - 3)/N$ (no, that's not the right answer). Everything in the sum needs to be clearly defined, but **YOU DO NOT NEED TO SIMPLIFY OR EVALUATE THE SUM.**