Name:

M340L Midterm Exam
October 12, 1993

Problem 1:

a) Is the matrix $A = \begin{pmatrix} 4 & -1 & 3 \\ 2 & 2 & 1 \\ 5 & 1 & 2 \end{pmatrix}$ singular or non-singular? Note that you do NOT have to compute A^{-1}.

b) Let $B = \begin{pmatrix} 4 & 1 & 5 \\ 3 & 1 & 2 \\ 7 & 2 & 6 \end{pmatrix}$. **USING MINORS**, compute B^{-1}. You MUST show your work.

Problem 2 This problem concerns vectors in $\mathbf{R}^3$.

a) Let $X = (3, 4, 5)$ and let $Y = (-4, 3, 5)$. Compute $\|X\|$, $\|Y\|$, and the angle between X and Y.

b) Find the area of the parallelogram with vertices at (1,2,4), (3,1,5), (5,2,6), and (7,1,7).

Part c is a watered-down version of part b. If you did part b, DO NOT DO PART c! If you CAN'T do part b, then you can do part c INSTEAD, for partial credit.

c) Find the area of the triangle with vertices at (1,2,4), (3,1,5) and (5,2,6).

Problem 3. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 3 & 1 & 4 & 2 & 6 \\ 1 & 3 & 4 & 2 & -2 \end{pmatrix}$

a) What is the rank of A?

b) What is the nullity of A (i.e. the dimension of the null space)?

c) Find a basis for the null space of A.

Problem 4.

a) Are the vectors $X_1 = (3, 2, 1)$, $X_2 = (1, 2, 3)$ and $X_3 = (1, 1, 1)$ linearly independent? If not, express one of them as a linear combination of the others.

b) Do the vectors $Y_1 = (1, 2, 4)$, $Y_2 = (2, 0, 2)$ and $Y_3 = (-1, 1, 4)$ span $\mathbf{R}^3$? If not, find a vector Y_4 that is not in the span of Y_1, Y_2, and Y_3. If they DO span $\mathbf{R}^3$, express $B = (0, 0, 1)$ as a linear combination of Y_1, Y_2, and Y_3.

Problem 5. Which of the following sets are vector spaces? No justification is needed.

a) All points (x_1, x_2, x_3) in $\mathbf{R}^3$ such that $x_1 + x_2 - 3x_3 \geq 0$.

b) All solutions to $AX = 0$, where A is a singular 5×5 matrix.

c) All vectors in $\mathbf{R}^4$ of the form $(a, a-b, 3a+b, b+1)$.

d) All linear combinations of the vectors (0,1,2,3), (3,2,1,0), and (1,1,1,1).

e) All vectors X in $\mathbf{R}^3$ such that $X \cdot Y = 2$, where $Y = (1, 2, 3)$.

f) All vectors in $\mathbf{R}^4$ with integer coefficients.

h) All points (x_1, x_2, x_3) in $\mathbf{R}^3$ such that $x_1 + x_2 - 3x_3 = 0$.

i) All solutions to $AX = B$, where A is a non-singular 5×5 matrix and $B = (1, 0, 0, 0, 0)$.