M362K Final Exam. May 10, 2002

Problem 1. Final Exams

The final exam for a certain math class is graded pass/fail. A randomly chosen student from a probability class has a 40% chance of knowing the material well. If he knows the material well, he has an 80% chance of passing the final. If he doesn't know the material well, he has a 40% chance of passing the final anyway.

a) What is the probability of a randomly chosen student passing the final exam?

b) If a student passes, what is the probability that he knows the material?

Problem 2. Hot Pickups

There are 123,850 pickup trucks in Austin. Of these, 2,477 are stolen. [OK, I'm making these numbers up.] Suppose that 100 randomly chosen pickups are checked by the police.

a) What is the probability that exactly 3 of the 100 chosen pickups are stolen. Give an EXACT answer, but do not evaluate it numerically.

b) Approximate this using the binomial distribution. What are n and p?

c) Approximate your answer to (b) using the Poisson distribution. What is λ?

Problem 3. Four Flushers

a) In poker, a "four-flush" is a hand with 4 cards of one suit, and one card of another suit. (This sort of hand isn't worth anything, and can be very frustrating). Find the probability of a (5-card) hand being a four-flush.

b) A "three-flush" is a hand with 3 cards of one suit, one of another suit and one of a third suit. (For instance, 3 spades, one heart and one club. A hand with 3 spades and 2 diamonds would not count.) Find the probability of a three-flush.

Problem 4. Joint distributions (2 pages)

X and Y are continuous random variables with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 1/2 & \text{if } 0 < x,\ 0 < y \text{ and } x + y < 2 \\ 0 & \text{otherwise} \end{cases}$$

a) Let $Z = X + Y$ Find the pdf of Z.

b) Find the pdf of X (that is, $f_X(x)$) and that of Y.

c) Find the expectation and variance of X.

d) Find the covariance $Cov(X, Y)$.

Problem 5. Quidditch

A game of Quidditch lasts until a player catches the Golden Snitch, which can happen at any time. Meanwhile, both teams try to score goals. The scoring by the two teams is described by independent Poisson processes. The Gryffindor team scores an average of 2.4 goals per hour, while the Slytherin team scores an average of 2.0 goals per hour.

A certain match between Gryffindor and Slytherin lasts 20 hours.

a) What is the probability that Slytherin doesn't score in the first *half* hour?

b) What is the probability that Gryffindor scores two or more goals in the first hour? Evaluate this answer numerically.

c) Using the normal approximation, estimate the probability that 100 or more goals (total) are scored in the entire match.

Extra Credit) What is the probability that Gryffindor scores more goals than Slytherin? Again, please give a numerical answer.

Problem 6. Change for a dollar

When I go grocery shopping, I pay a whole number of dollars, and get back some coins in change. Suppose that the value of these coins is uniformly distributed between 0 and 1 (dollars). [In reality, the change is a discrete random variable, with possible values of 0, 0.01, 0.02, up to 0.99. However, we'll approximate it by a continuous variable that ranges from 0 to 1.] I go shopping many times, and the change I get on the different days is denoted X_1, X_2, etc. You may assume that these variables are independent.

a) Let $Y = X_1 + X_2$. Find the pdf $f_Y(y)$ for all values of y.

b) Let $S = X_1 + X_2 + \cdots + X_{48}$. Find the expectation and variance of S.

c) Estimate the probability that $Y > 25$.

Problem 7. Parking

A typical Austin parking garage charges \$1 for each hour you park, or portion thereof. That is, if you park for less than an hour, you pay \$1, if you park for more than an hour but less than two hours you pay \$2, and so on.

Let X be the number of hours that a randomly chosen car is in the garage. The distribution of X is exponential (that is, $f(x)$ is of the form $(1/\lambda)e^{-x/\lambda}$ for $x > 0$) with a mean of 3 hours.

a) What is the probability that $1 \leq X \leq 3$?

b) What is the probability that a random car's driver will be charged \$2?

c) Let Y be the cost of parking a car. Is this a continuous or discrete random variable? What are the possible values of Y? Find the pdf $f_Y(y)$.