M362K Final Exam. May 13, 2004

Problem 1. Delusional politicians

1% of the population suffers from "delusions of grandeur", a person's belief that he is far, far more important than the facts would indicate. 5% of the people with such delusions run for public office, but only 0.1% of people without delusions of grandeur run for public office. [Note: these numbers are made up, but the psychosis is real. I'll leave it to you to decide whether the diagnosis applies to anybody you know.]

a) What is the probability of a randomly chosen person running for office?

b) Given that a person is running for office, what is the probability of his having delusions of grandeur?

Problem 2. 5 Crowns

5 Crowns is a real card game that my children enjoy. There are 5 suits (clubs, diamonds, hearts, spades and stars) and the cards range in value from 3 to King (no aces or 2s). Suppose a player is dealt 8 cards.

a) What is the probability that he has no kings? Give an exact answer, which you can leave in the form of factorials or binomial coefficients.

b) What is the probability that he has 3 clubs, 2 diamonds, one heart, one spade and one star?

c) What is the probability that he has 4 pair?

Extra credit (write answer on back): Approximate the answer to (a) using the binomial or Poisson distribution. Your final answer should be numerical.

Problem 3. Joint distributions

X and Y are discrete random variables whose joint pdf is given in the table :

$Y\backslash X$	1	2	5
1	.12	.03	.15
2	.20	.05	.25
3	.08	.02	.10

a) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$ for all possible values of X and Y, respectively.

b) Find the conditional probability $P(Y = 3|X = 2)$. Are the events "$Y = 3$" and "$X = 2$" independent? Are X and Y independent random variables?

c) Let $Z = X + Y$. Compute the pdf $f_Z(z)$.

Problem 4. Light bulbs

The lifetime of a 60 watt incandescent light bulb is given by the exponential distribution, with mean 1000 hours.

a) Find the probability that a randomly chosen light buld last less than 50 hours.

b) A Christmas light display consists of 1000 light bulbs, whose lifetimes are assumed to be independent. Let X be the number of light bulbs that burn out in the first 50 hours. Which distribution describes X? Give an exact formula for $f_X(x)$.

c) Approximate the probability that $X \leq 45$ using the normal distribution.

Problem 5. Misbehaving kids

Like all children, my kids sometimes misbehave. Being children of a mathematician, they misbehave according to independent Poisson processes. Allan misbehaves an average of 3 times per hour, Rina an average of 2 times per hour, and Jonathan an average of 1.5 times per hour.

a) In one hour, what is the probability that Jonathan doesn't misbehave at all? What is the probability that Allan misbehaves 2 or fewer times.

b) In a 15 minute period, what is the probability that there will be exactly one instance of misbehavior among the kids?

c) Let X be the total number of misbehaviors in 2 hours (for all three kids put together). Find the mean and standard deviation of X.

Problem 6. Manipulating random variables (10 points)

Let X be a continuous random variable, uniformly distributed between 1 and 2. Let $Y = X^2$.

a) Find $f_Y(y)$ for all values of y.

b) Find $E(Y)$ and $Var(Y)$. [Note: it is possible to do (b) without first doing (a).]

Problem 7. Pennies

A penny is approximately 1 gram. That is, the weight of a (random) penny is a continuous random variable with mean 1.00 grams and standard deviation 0.04 grams. A roll of pennies contains 50 pennies, whose weights are independent. Let X be the weight of a roll of pennies, in grams.

a) Find $E(X)$ and $Var(X)$.

b) Using the normal distribution, estimate the probability that $49.5 < X < 50.5$.