

Problem 1. Sample spaces and events

In the game of “third chance” (which I just made up), a player rolls an ordinary 6-sided die. If the die comes up 1 or 2, he rolls a second die. If the second die comes up 1, he rolls a third die. His score is the sum of all the dice rolled. For instance, he might score a 5 by rolling a 5 on his first throw, by rolling a 1 and then a 4, by rolling a 2, then a 1, and then a 2, or by several other means. [To fix notation once and for all, let’s describe those three scenarios as (5), (1,4) and (2,1,2)]. Let A be the event “player rolls exactly 2 dice” and let B be the event “player scores 6”.

- List all the points in the sample space. How many are there?
- List the points in the event A . Separately, list the points in the event B .
- List the points in the event $A \cap B$. List the points in the event $A \cup B$.

Problem 2. Continuous distributions

A random number is chosen from the interval $0 \leq x \leq 1$ with probability density function $f(x) = 2x$. Let A be the event “number is between $1/3$ and $2/3$ ”, and let B be the event “number is greater than $1/2$ ”.

- Find the probability of A
- Find the probability of B .
- Find the conditional probability $P(A|B)$.

Problem 3. Independence

A gambler rolls two fair dice (one red and one green). Let A be the event “the total roll is a 6 or an 8”, let B be the event “The red die shows a 1, 2 or 3”, and let C be the event “The green die shows a 1, 2, or 3”.

- Compute the probabilities of A , B , C , $A \cap B$, $A \cap C$, $B \cap C$ and $A \cap B \cap C$.
- Are A and B independent events? Are A and C independent events? Are B and C independent events?
- Are A , B and C independent events?

Problem 4. Stolen elections

In the race for Sivart County Commissioner, the candidate of the Loot and Pillage (L&P) Party has hired some hackers to break into the election computers and fix the result. The hackers have a 20% chance of breaking into the system. If they get in, they have a 90% chance of stealing enough votes to win the election. If they don’t get in, the L&P candidate has a 15% chance of winning anyway.

- What is the probability that the L&P candidate will win the election.
- Suppose that the L&P candidate wins. What is the probability that the hackers succeeded in breaking into the system?