M340L Midterm Exam
September 16, 1993

Problem 1: Find all solutions to the following system of equations:

$$\begin{aligned} x_1 + \quad x_2 + \quad x_3 - 3x_4 &= 6 \\ 2x_1 + 3x_2 - \quad x_3 - 4x_4 &= 11 \\ x_1 - \quad x_2 + \quad x_3 - \quad x_4 &= 2 \\ x_1 - \quad x_2 - \quad x_3 + \quad x_4 &= 0 \end{aligned}$$

Problem 2

a) Is the matrix $A = \begin{pmatrix} 1 & 6 & 3 \\ 1 & 5 & 2 \\ 0 & 2 & 1 \end{pmatrix}$ singular or non-singular? If A is non-singular, find A^{-1}.

b) Find all solutions to $AX = B$, where A is given above and $B = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$. (Hint: Use the result of part a)

Problem 3. By doing row operations, put these matrices in reduced row-echelon form:

a)

$$\begin{pmatrix} 1 & 2 & 1 & 4 & 3 \\ 2 & 4 & 3 & 6 & 5 \\ -1 & -2 & 0 & 2 & 12 \end{pmatrix}$$

b)

$$\begin{pmatrix} 0 & 2 & 4 \\ -2 & 0 & 2 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{pmatrix}$$

Problem 4. Evaluate the following determinants:

a)

$$\begin{vmatrix} 3 & 5 \\ 7 & 12 \end{vmatrix}$$

b)

$$\begin{vmatrix} 1 & 3 & 2 \\ -1 & 1 & 5 \\ 2 & -2 & 4 \end{vmatrix}$$

Problem 5. True of False

a) If A is a 3×5 matrix that has rank 3, then the equation $AX = B$ has at least one solution, regardless of what B is.

b) If A is a 5×3 matrix that has rank 3, then the equation $AX = B$ has at least one solution, regardless of what B is.

c) If A is a singular $n \times n$ matrix, then $AX = 0$ has infinitely many solutions.

d) If A is a nonsingular $n \times n$ matrix, then $AX = B$ has exactly one solution, namely $X = A^{-1}B$.

e) 5 2 1 4 3 is an even permutation of 1 2 3 4 5

f) If A and B are nonsingular $n \times n$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.

g) If $AX = B$ has exactly one solution, then $AX = 0$ has exactly one solution.

h) If $AX = 0$ has infinitely many solutions, then $AX = B$ has infinitely many solutions.

i) If $AX = B$ has infinitely many solutions, then $AX = 0$ has infinitely many solutions.

j) If two rows of a square matrix are the same, the determinant of that matrix is zero.