M340L Midterm Exam
April 11, 1995

Problem 1: Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & -2 \\ 3 & 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 3 & 1 \end{pmatrix}$$

a) Compute $\det(A)$ and $\det(B)$.

b) Is A invertible? If so, compute $\det(A^{-1}B)$. If not, find a nonzero vector in $Nul(A)$. Note: There is an easy way to do this part of the problem.

Problem 2

Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 & 3 \\ 2 & 4 & 3 & 3 & 7 \\ 2 & 4 & 4 & 6 & 8 \end{pmatrix}$$

a) What is the rank of A?

b) What is the dimension of the null space of A? The row space?

c) Find a basis for the column space of A.

d) Find a basis for the null space of A.

Problem 3. Let $\mathcal{D} = \{d_1, d_2, d_3\}$ and $\mathcal{F} = \{f_1, f_2, f_3\}$ be bases for a 3-dimensional vector space V, and suppose that $f_1 = 2b_1 - b_2 + b_3,\ f_2 = 3b_2 + b_3,\ f_3 = 4b_1 + 2b_3$.

a) Find the change-of-coordinates matrix from $\mathcal{F}$ to $\mathcal{D}$.

b) Find the change-of-coordinates matrix from $\mathcal{D}$ to $\mathcal{F}$.

c) If $[x]_{\mathcal{F}} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, find $[x]_{\mathcal{D}}$.

d) If $[y]_{\mathcal{D}} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$, find $[y]_{\mathcal{F}}$.

Problem 4. Let $A = \begin{pmatrix} -3 & -2 \\ 3 & 4 \end{pmatrix}$.

a) Find the eigenvalues of A.

b) For each eigenvalue, find a corresponding eigenvector.

Problem 5. True of False

a) If A and B are square and invertible, then AB is invertible and $(AB)^{-1} = A^{-1}B^{-1}$.

b) If 2 rows of a 3×3 matrix A are the same, then $\det(A) = 0$.

c) If A is a 4×4 matrix, then $\det(4A) = 4\det(A)$.

d) If A is invertible, then $\det(A) \neq 0$.

e) If A is a square stochastic matrix, then $A - I$ is not invertible.

f) If V is a 5-dimensional vector space, then there is a set of vectors $\{v_1, \ldots, v_6\}$ that spans V.

g) If V is 5-dimensional, then every collection of 4 vectors in V is linearly independent.

h) If there is a collection $\{v_1, \ldots v_7\}$ that spans a vector space V, then the dimension of V is at least 7.

i) If two matrices are row-equivalent, then their null spaces are the same.

j) If two matrices are row-equivalent, then their column spaces are the same.