Quadrics.

$$\begin{pmatrix} \lambda_1 x^2 \text{ or } b_1 x \\ \text{or } 0 \end{pmatrix} + \begin{pmatrix} \lambda_2 y^2 \text{ or } b_2 y \\ \text{or } 0 \end{pmatrix} + \begin{pmatrix} \lambda_3 z^2 \text{ or } b_3 z \\ \text{or } 0 \end{pmatrix} = c$$

---

2 0's.

$$\lambda_1 x^2 = c$$

$$x^2 = c/\lambda_1 = \begin{cases} \text{plane if } c=0 \\ \text{2 planes if } c/\lambda_1 > 0 \\ \emptyset \text{ if } c/\lambda_1 < 0 \end{cases}$$

$$b_1 x = c \Rightarrow x = \frac{c}{b_1} = \text{plane}$$

---

1 0

$$\lambda_1 x^2 + \lambda_2 y^2 = c \qquad \begin{pmatrix} \text{ellipse or hyperbola} \\ \text{or pt. or} \\ \text{2 lines} \end{pmatrix} \times \mathbb{R}$$

$$\lambda_1 x^2 + b_2 y = c \qquad \text{parabolic cylinder.}$$

$$b_1 x + b_2 y = c \qquad \text{plane.}$$

No 0's:

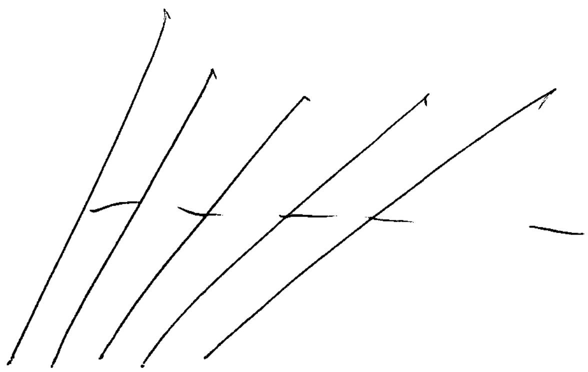
$\lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2 = c$ ellipsoid or pt. or hyperbaloid or cone.

$\lambda_1 x^2 + \lambda_2 y^2 + b_3 z = c$ paraboloid

$\lambda_1 x^2 + b_2 y + b_3 z = c$ parabolic cylinder

$b_1 x + b_2 y + b_3 z = c$ plane.

A ruled surface is a family of lines.



Family $\vec{\gamma}(u)$ of starting pts,
$\vec{\delta}(u)$ of vectors.

$$\sigma(u,v) = \vec{\gamma}(u) + v\,\vec{\delta}(u)$$

$$\sigma_u = \dot{\gamma} + v\,\dot{\delta} \qquad \dot{} = \frac{d}{du}$$

$$\sigma_v = \delta$$

$$\sigma_u \times \sigma_v = (\dot{\gamma} \times \delta) + v\,(\dot{\delta} \times \delta)$$

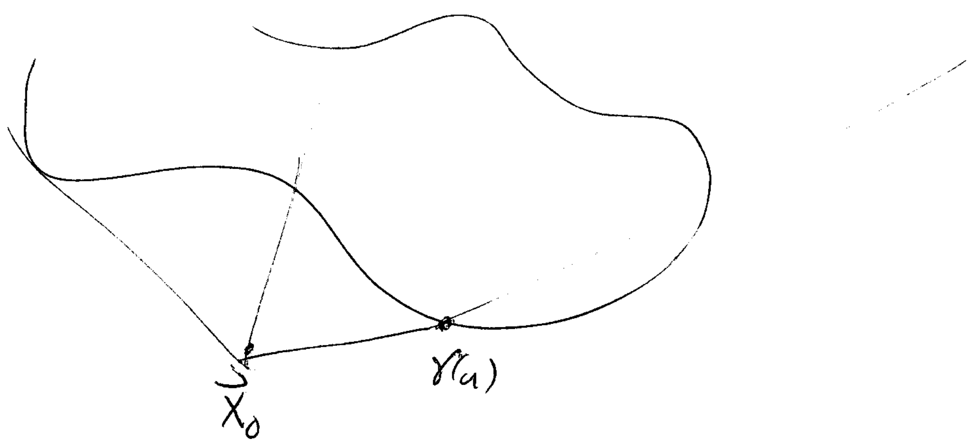
$\dot{\gamma} \times \delta$ should be $\neq 0$ and not a multiple of $\dot{\delta} \times \delta$

Cylinders

$\delta =$ constant $= \vec{a}$.

$$\sigma(u,v) = \gamma(u) + v\,\vec{a}$$
$$\sigma_u = \dot{\gamma} \quad \sigma_v = \vec{a}$$

Cone: $\delta(u) = \gamma(u) - \vec{x}_0$



$$\sigma_1(u,v) = \gamma(u) + v\,\delta(u) = \gamma(u) + v\,(\gamma(u) - x_0)$$
$$= \vec{\gamma}(u)(1+v) - v\vec{x}_0$$

$$z = xy \qquad \text{hyperbolic paraboloid.}$$

$$= \frac{(x+y)^2 - (x-y)^2}{4}$$

$$\gamma(u) = (u, 0, 0) \qquad \sigma(u,v) = \gamma(u) + v\,\delta(u)$$

$$\delta(u) = (0, 1, u) \qquad = (u, v, uv)$$

$$\tilde{\gamma}(u) = (0, u, 0)$$

$$\tilde{\delta}(u) = (1, 0, u) \qquad \tilde{\sigma}(u,v) = (v, u, uv)$$

Doubly ruled surface

Hyperbaloid. $x^2+y^2-z^2=1$

$\gamma(u) = (\cancel{\sin(u)}\ \cos(u), \sin(u), 0)$

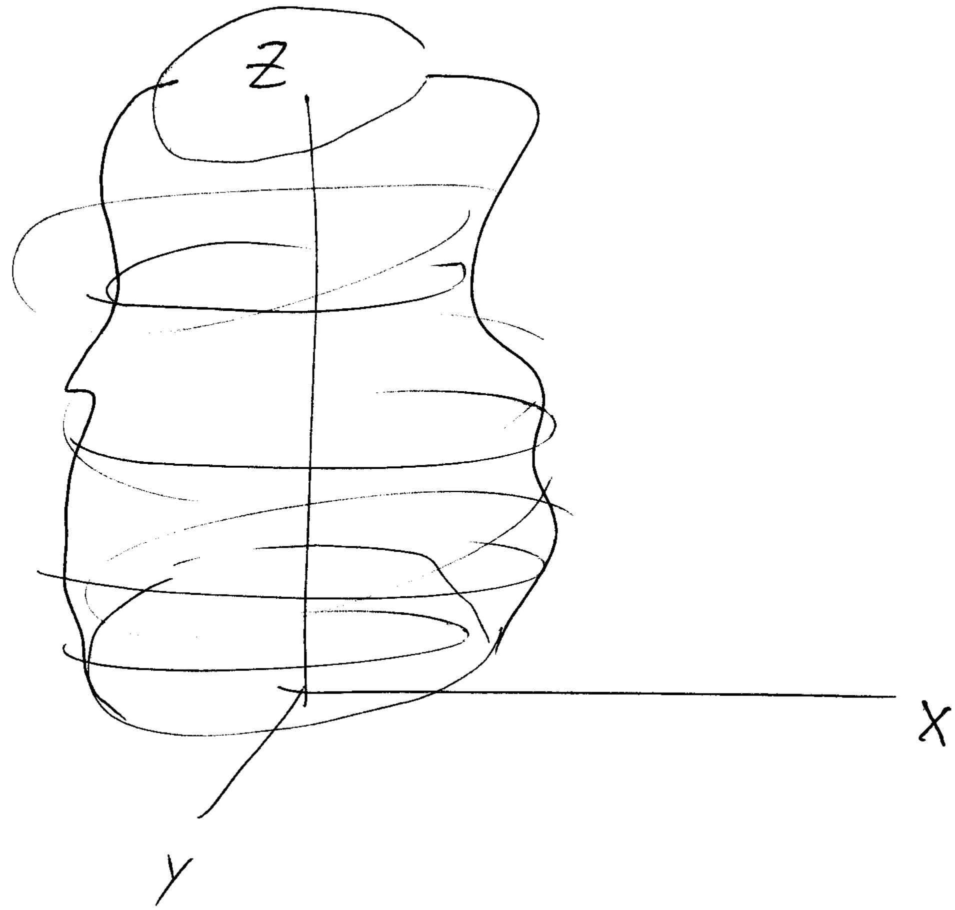
$\delta(u) = \dot{\gamma}(u) \pm (0,0,1) = (-\sin(u), \cos(u), \pm 1)$

$\sigma(u,v) = \gamma(u) + v\,\delta(u) = \gamma(u) + v\,\dot{\gamma}(u) + (0,0,v)$

$x^2+y^2 = |\gamma(u)|^2 + v^2\,|\dot{\gamma}(u)|^2 + 2v\,(\gamma\cdot\dot{\gamma}) = 1+v^2$

$z^2 = v^2$

$x^2+y^2-z^2=1$



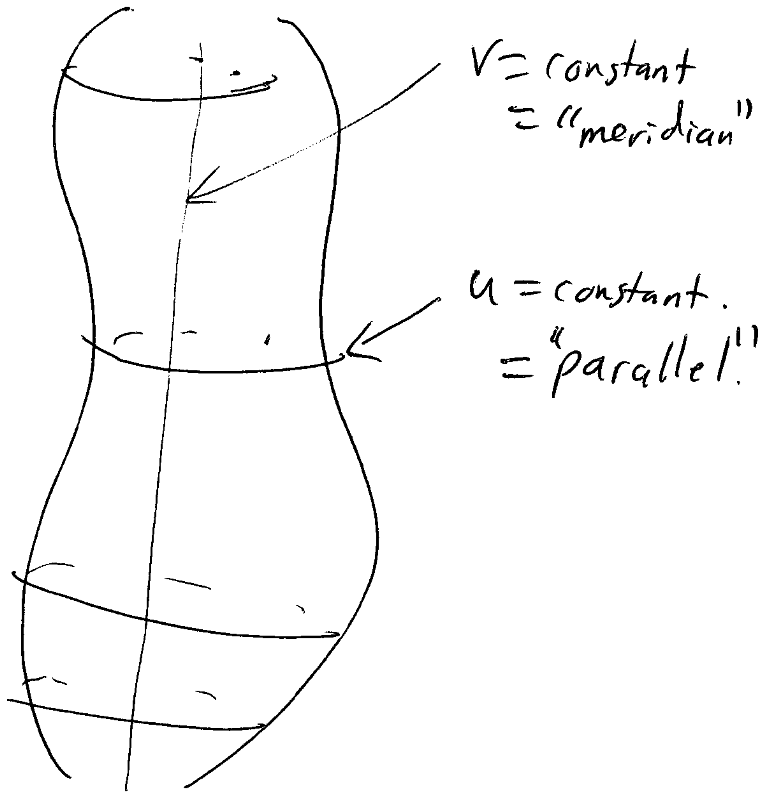
A surface of revolution is obtained by rotating a plane curve about an axis in that plane.

Curve $\gamma(u) = (f(u), 0, g(u))$ rotate by v to get

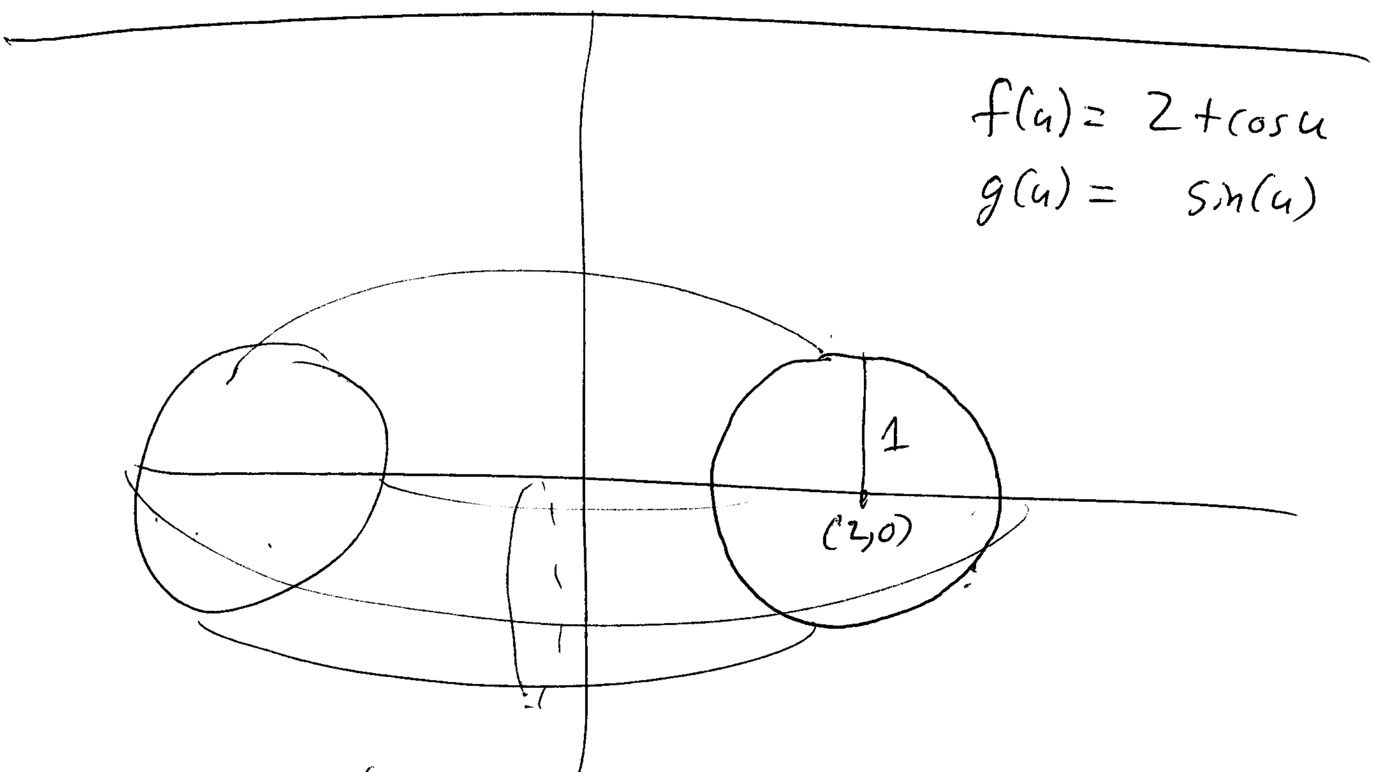
$$\sigma(u,v) = (f(u)\cos v, f(u)\sin v, g(u))$$

$$\sigma_u = (\dot{f}\cos v, \dot{f}\sin v, \dot{g})$$

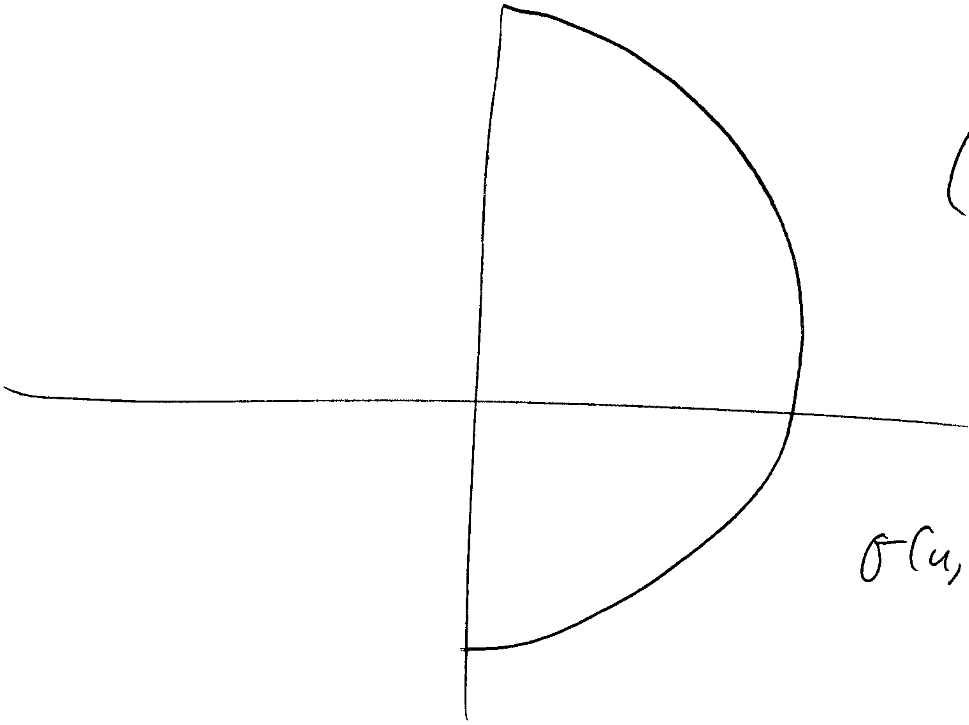
$$\sigma_v = (-f\sin v, f\cos v, 0)$$



$$f(u) = 2 + \cos u$$
$$g(u) = \sin(u)$$



$$\sigma(u,v) = \big((2+\cos u)\cos v,\ (2+\cos u)\sin(v),\ \sin(u)\big)$$



$(\cos u, 0, \sin(u))$

$$\sigma(u,v) = (\cos u \cos v, \cos u \sin v, \sin u)$$

Latitude-longitude description of sphere.

A surface is compact if it is closed & bounded.

S^2 is compact.

$N = \{ (x,y,z) \mid x^2+y^2+z^2=1 \text{ and } z>0 \}$.

isn't closed.

$\overline{N} = \{ (x,y,z) \mid x^2+y^2+z^2=1 \text{ and } z \geq 0 \}$.

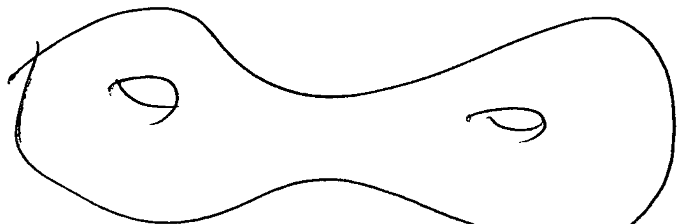
Closed, bounded, compact, but not a surface.

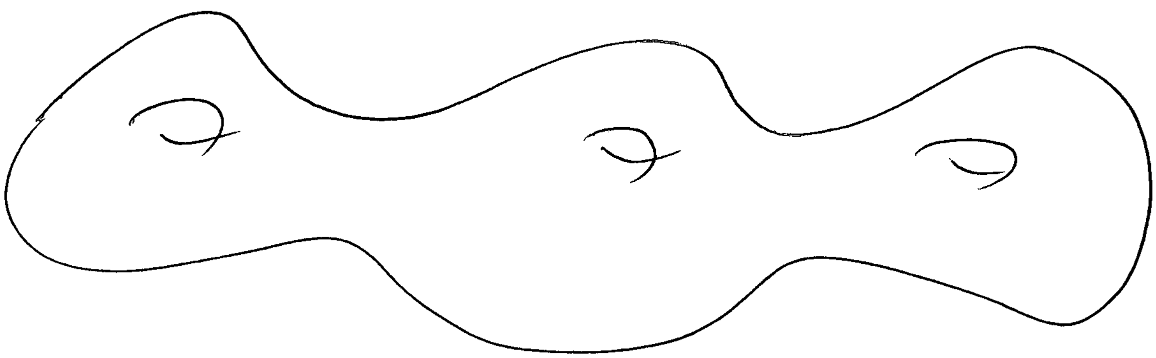
x-y plane. Closed, not bounded.

Examples of compact surfaces:

1) S^2

2) $T^2 =$ 

3) 

4) 

T_g = g-hole torus (get by sticking g unbaked bagels together)

Anything diffeomorphic to $\underbrace{S^2}_{T_0 =}$, $\underbrace{T^2}_{\cong T_1}$, T_g.

Thm

That's all she wrote.

Every compact surface in $\mathbb{R}^3$ is diffeo to T_g for some $g \geq 0$.