Thm If B is an $n\times n$ real symmetric matrix, then ~~e-val~~

1) B is diagonalizable
2) e-vals of B are real
3) e-vecs can be chosen $\perp$.

Thm If A, B are real symmetric, $A>0$ ~~then~~

1) $A^{-1}B$ is diagonalizable
2) e-vals are real
3) e-vecs are A-$\perp$.

If $A^{-1}B\vec{x} = \lambda_1 \vec{x}$, $A^{-1}B\vec{y} = \lambda_2 \vec{y}$ and $\lambda_1 \neq \lambda_2$

then $X^TAy = 0$

pf of (3): $\cancel{X^TAy = X^TA}$

$$X^TBy = X^TAA^{-1}By = X^TA\lambda_1 y = \lambda_1(X^TAy)$$

$$y^TBx = \lambda_2(y^TAx)$$

$\cancel{\lambda_1}(\lambda_1 - \lambda_2)(X^TAy) = 0 \Rightarrow X^TAy = 0$

<u>Cor</u> E-vecs of W are $\perp$.

Pick e-vecs $\vec{V}_1, \vec{V}_2$ orthonormal,

$\dot{\gamma} = \cos\theta\, \vec{V}_1 + \sin\theta\, \vec{V}_2$,

$\langle\langle \dot{\gamma}, \dot{\gamma} \rangle\rangle = K_1 \cos^2\theta + K_2 \sin^2\theta$.

$H = \frac{K_1 + K_2}{2}$ = average normal curvature of curves cut out by planes $\perp$ S.

If $K_1(p)$ and $K_2(p)$ have same sign, think: paraboloid

$K>0$ elliptic point.

If ~~$K_1 \neq 0$, K~~ opposite signs, hyperbolic.
$K<0$

If $K_1 = 0$, $K_2 \neq 0$, parabolic (think: cylinder)

$K=0$

If $K_1 = K_2 = 0$ planar.

Surface locally looks like

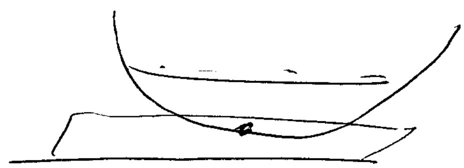
$$Z = \frac{K_1}{2}X^2 + \frac{K_2}{2}Y^2 + \text{higher order.}$$

If $K_1 = K_2$, all directions look same.

umbilical) point.

Thm Every compact surface has a point with $K \geq 0$.

pf Find pt with $z =$ minimum.



θ slice w/ plane, curve curves up

$$\kappa_1 \cos^2\theta + \kappa_2 \sin^2\theta \geq 0 \quad \text{for all } \theta$$

$$\kappa_1, \kappa_2 \geq 0 \quad , \quad \kappa_1 \kappa_2 \geq 0.$$

Curve in x-z plane $\gamma(u) = (f(u), 0, g(u))$

$|\dot\gamma|^2 = \dot f^2 + \dot g^2 = 1$

rotate about z-axis

$\sigma(u,v) = (f(u)\cos v,\ f(u)\sin v,\ g(u))$

$\sigma_u = (\dot f \cos v,\ \dot f \sin v,\ \dot g)$

$\sigma_v = (-f \sin v,\ f\cos v,\ 0)$

$E = \dot f^2 + \dot g^2 = 1$

$F = 0$

$G = f^2$

$\vec N = \dfrac{\sigma_u \times \sigma_v}{f}$

$$\begin{vmatrix} i & j & k \\ \dot f\cos v & \dot f \sin v & \dot g \\ -\sin v & \cos v & 0 \end{vmatrix} = (-\dot g\cos v,\ -\dot g \sin v,\ \dot f)$$

$\sigma_{uu} = (\ddot f \cos v,\ \ddot f \sin v,\ \ddot g)$

$\sigma_{uv} = (-\dot f \sin v,\ \dot f\cos v,\ 0)$

$\sigma_{vv} = (-f\cos v,\ -f\sin v,\ 0)$

$L = -\dot g \ddot f + \dot f \ddot g$

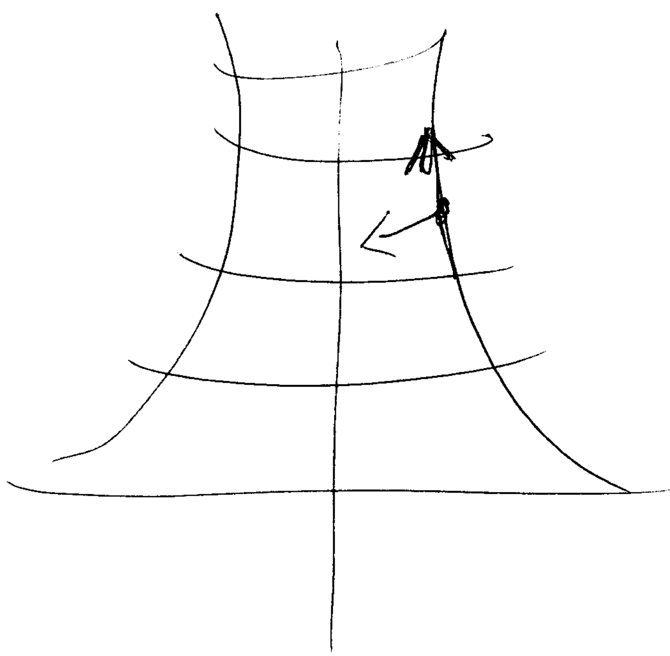
$M = 0$

$N = f\dot g$

$$\mathrm{I} = \begin{pmatrix} E & F \\ F & G \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & f^2 \end{pmatrix}$$

$$\begin{pmatrix} L & M \\ M & N \end{pmatrix} = \begin{pmatrix} -\dot{g}\ddot{f} + \dot{f}\ddot{g} & 0 \\ 0 & f\dot{g} \end{pmatrix}$$

$$W = \begin{pmatrix} -\dot{g}\ddot{f} + \dot{f}\ddot{g} = \kappa_1 & 0 \\ 0 & \dot{g}/f \end{pmatrix} = \kappa_2$$



$$K = \left(-\dot{g}\ddot{f} + \dot{f}\ddot{g}\right)\frac{\dot{g}}{f}$$

$$= \frac{\left(-\dot{g}^2\ddot{f} + \dot{f}\dot{g}\ddot{g}\right)}{f} = \frac{-\dot{g}^2\ddot{f} - (\dot{f})^2\ddot{f}}{f}$$

$$= -\ddot{f}/f$$

$$\dot{f}^2 + \dot{g}^2 = 1 \qquad \dot{f}\ddot{f} + \dot{g}\ddot{g} = 0$$

$$K = 1 \Rightarrow -\frac{\ddot{f}}{f} = 1 \Rightarrow \ddot{f} + f = 0$$

$$f = a\cos(u)$$

$$\dot{f} = -a\sin(u)$$

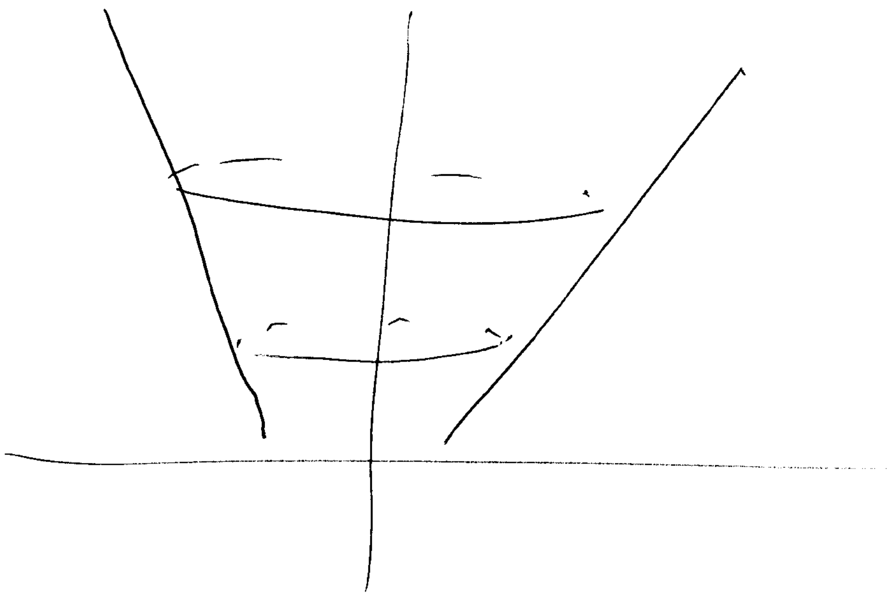
$$\dot{f}^2 = a^2\sin^2 u$$

$$\dot{g}^2 = 1 - a^2\sin^2 u$$

$$g = \int\sqrt{1 - a^2\sin^2 u}\,du$$

$K = 0$ $\quad$ $\ddot{f} = 0$

$\dot{f} = \text{const}$ $\quad$ $\dot{g} = \text{const.}$



$K = -1.$ $\quad$ $\ddot{f} = f$

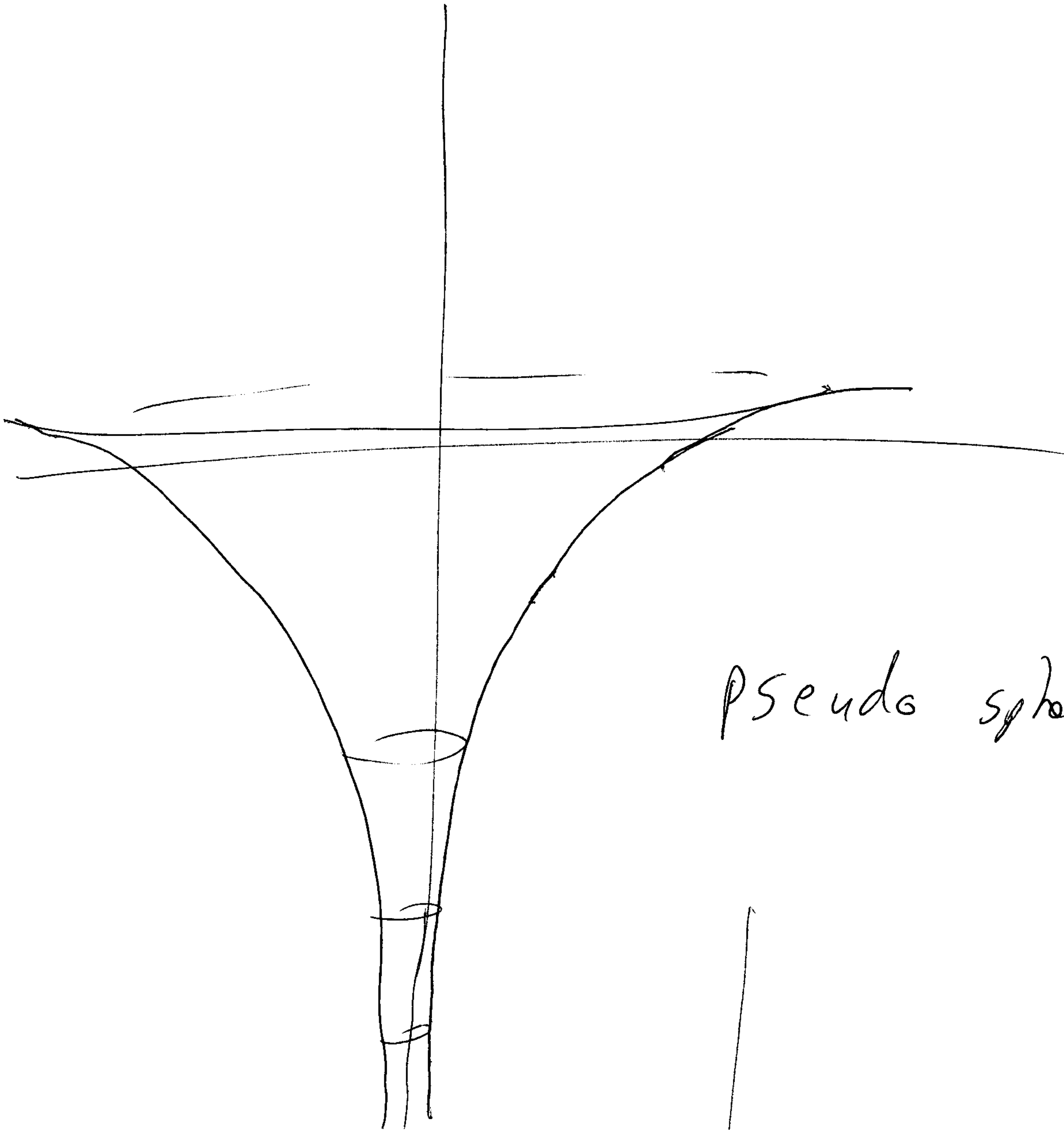
$$f = c_1 e^{t} + c_2 e^{-t}$$

Look at case $c_1 = 1$, $c_2 = 0$.

$$\dot{f} = c_1 e^{t}$$

$$(\dot{f})^2 = c_1^2 e^{2t}$$

$$(\dot{g})^2 = 1 - c_1^2 e^{2t}$$



pseudo sphere.