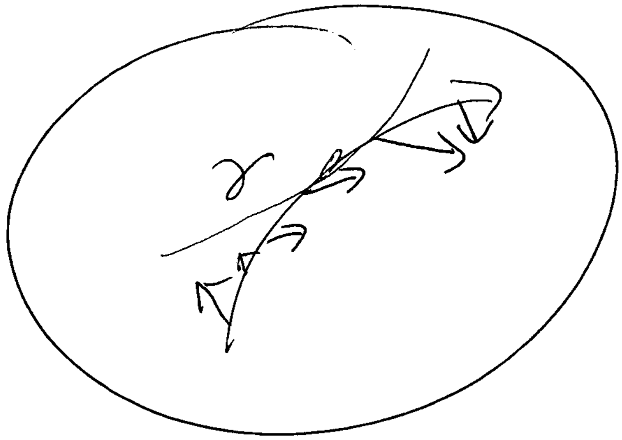


Covariant derivative $\nabla_\gamma \vec{W}$ = tangential part of $(\dot{\vec{W}})$

Depends only on $\dot{\gamma}$, can make into directional derivative.

$$\nabla_i e_j = \sum_k \Gamma_{ij}^k e_k$$

$$\Gamma_{ij}^k = \text{Christoffel symbol.} = \Gamma_{ji}^k$$

$\nabla_i e_j$ = horizontal part of σ_{ij} = horizontal part of σ_{ji}

$= \nabla_j e_i$

$$\sigma_{uu} = \underbrace{\Gamma^1_{11}\sigma_u + \Gamma^2_{11}\sigma_v}_{\nabla_1 e_1} + L\vec{N}$$

$$\left.\begin{aligned}
\sigma_{uv} &= \Gamma^1_{12}\sigma_u + \Gamma^2_{12}\sigma_v + M\vec{N} \\
\sigma_{vu} &= \Gamma^1_{21}\sigma_u + \Gamma^2_{21}\sigma_v + M\vec{N} \\
\sigma_{vv} &= \Gamma^1_{22}\sigma_u + \Gamma^2_{22}\sigma_v + N\vec{N}
\end{aligned}\right\}\ \text{Gauss equations}$$

$$\begin{pmatrix} \Gamma^1_{11} \\ \Gamma^2_{11} \end{pmatrix} = \begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} E_u/2 \\ F_u - E_v/2 \end{pmatrix}$$

$$\begin{pmatrix} \Gamma^1_{12} \\ \Gamma^2_{12} \end{pmatrix} = \begin{pmatrix} \Gamma^1_{21} \\ \Gamma^2_{21} \end{pmatrix} = \begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} E_v/2 \\ G_u/2 \end{pmatrix}$$

$$\begin{pmatrix} \Gamma^1_{22} \\ \Gamma^2_{22} \end{pmatrix} = \begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} F_v - \frac{G_u}{2} \\ G_v/2 \end{pmatrix}$$

$\vec{W}$ is parallel if

$x^1 = u$

$x^2 = v$

$$\dot{w}^i + \sum_{jk} \Gamma^i_{jk} \dot{x}^j w^k = 0$$

$\dot{\gamma}$ is parallel if

$$\boxed{\ddot{x}^i + \sum_{jk} \Gamma^i_{jk} \dot{x}^j \dot{x}^k = 0}$$ Geodesic.

$\nabla_\gamma \dot{\gamma} = 0$

Sphere, $u = \theta = \cancel{\phi}$ latitude

$v = \phi =$ longitude

$\cancel{\sigma(u,v) =}$

$\sigma(\theta,\phi) = (\cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta)$

$E = 1$

$F = 0$

$G = \cos^2\theta$

$G_u = G_\theta = -2\sin\theta\cos\theta$

all other derivatives $= 0$.

$$\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & \sec^2\theta \end{pmatrix}$$

$\Gamma_{11}^1 = \Gamma_{11}^2 = 0$

$$\begin{pmatrix} \Gamma_{12}^1 \\ \Gamma_{12}^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \sec^2\theta \end{pmatrix} \begin{pmatrix} 0 \\ -\sin\theta\cos\theta \end{pmatrix}$$

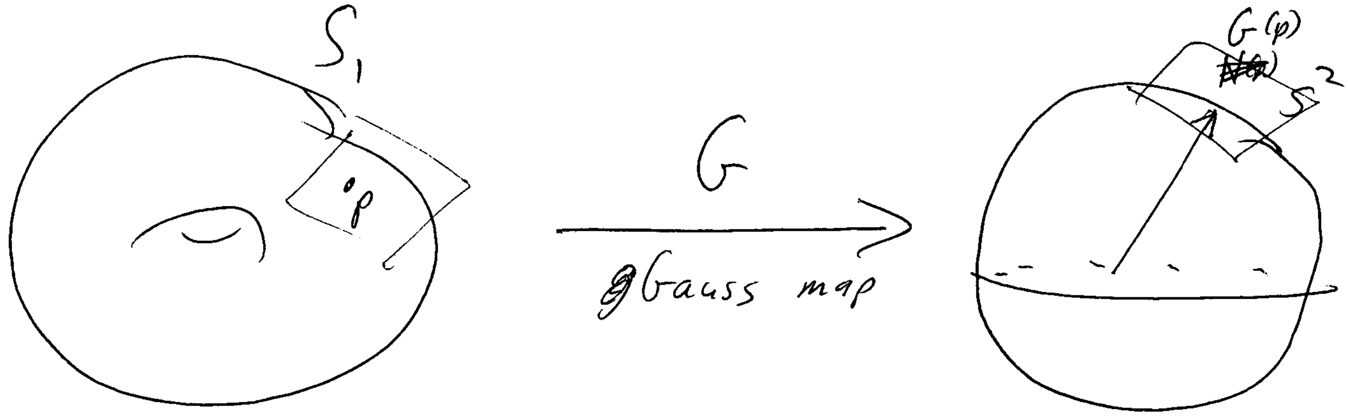
$$\begin{pmatrix} \Gamma_{22}^1 \\ \Gamma_{22}^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \sec^2\theta \end{pmatrix} \begin{pmatrix} \sin\theta\cos\theta \\ 0 \end{pmatrix}$$

$\Gamma_{12}^2 = \Gamma_{21}^2 = -\tan\theta$

$\Gamma_{22}^1 = \sin\theta\cos\theta$

$$\ddot{\theta} + \sin\theta\cos\theta\,(\dot{\phi})^2 = 0$$

$$\ddot{\phi}\ \cancel{\theta} - 2\tan\theta\,\dot{\theta}\,\dot{\phi} = 0$$



$$G(p) = \vec{N}(p)$$

$$T_p S_1 = \text{all vectors in } \mathbb{R}^3 \perp \text{ to } \vec{N}(p)$$

$$= \text{" " " "} \perp \text{ to } G(p)$$

$$= T_{G(p)} S^2$$

$$dG : T_p S_1 \longrightarrow T_{G(p)} S^2 = T_p S_1$$

$$W = -dG$$

$$W(w) = -\partial_{\vec{w}} \vec{N} = -w^1 \partial_u N - w^2 \partial_v N$$

$$\langle\!\langle \vec{w}, \vec{z} \rangle\!\rangle = W(\vec{w}) \cdot \vec{z}$$

Matrix of W is $\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} L & M \\ M & N \end{pmatrix}$

$$= \frac{1}{EG-F^2} \begin{pmatrix} G & -F \\ -F & E \end{pmatrix} \begin{pmatrix} L & M \\ M & N \end{pmatrix}$$

$$= \frac{1}{EG-F^2} \begin{pmatrix} LG-FM & MG-FN \\ EM-FL & EN-FM \end{pmatrix}$$

$$K = \det W = \frac{LN-M^2}{EG-F^2} = \cancel{\kappa_1+1}\ \kappa_1 \kappa_2$$

= Gaussian Curv

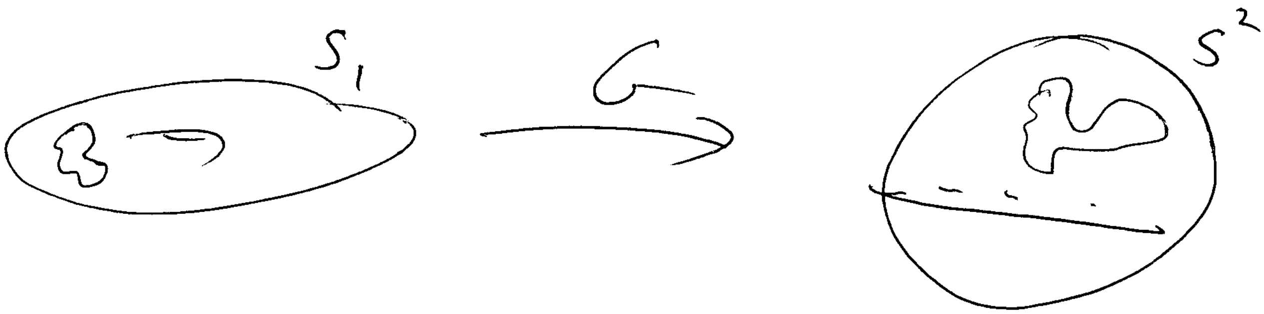
$$H = \frac{1}{2} \operatorname{Tr} W = \frac{LG-2FM+EN}{2(EG-F^2)} = \frac{\kappa_1+\kappa_2}{2}$$

= Mean curvature.

κ_1, κ_2 = e-vals of W

= ~~prpal~~ principal curvatures

$$\kappa_{1,2} = H \pm \sqrt{H^2 - K}$$

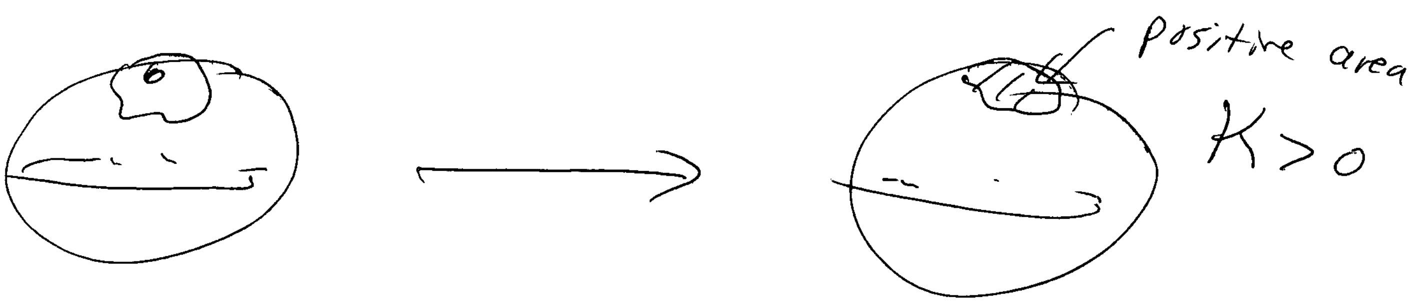


$$dG = -W$$

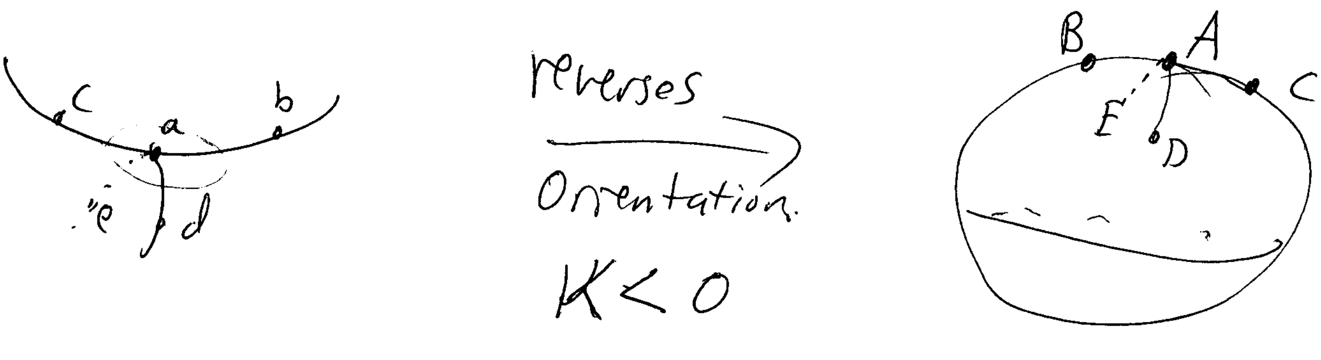
$$\det(dG) = \det W = K = \text{Gaussian curvature.}$$

$$= \pm \frac{(\text{area in } S^2)}{\text{area in } S^1}$$

$$\int_{\substack{\text{part}\\ \text{of } S_1}} K \, d(\text{area}) = \begin{pmatrix} \text{signed} \\ \text{area of image} \\ \text{in } S^2 \end{pmatrix}$$



$K > 0$



$K < 0$

$$Z = \frac{a}{2}x^2 + \frac{c}{2}y^2$$

$$W(0) = \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix} \qquad K = ac$$

Force from surface tension

$= (\text{constant}) \cdot H \cdot \vec{N}$

Soap bubble, Δpressure $\propto H$.

If pressure = constant, $H = 0$.

"Minimal surface"

If M is a symmetric real matrix,

e-vals are real

e-vecs are $\perp$.

If $M = A^{-1}B$ and A, B symmetric,

then e-vals of M are real, and

e-vecs have property that $\vec{x}_j^T A \vec{x}_i = 0$ for $i \neq j$

$$M = A^{-1/2} \underbrace{\left(A^{-1/2} B A^{-1/2}\right)} A^{1/2}$$

If $A^{-1/2} B A^{-1/2} \vec{x} = \lambda_1 \vec{x}$

$A^{-1/2} B A^{-1/2} \vec{y} = \lambda_2 \vec{y}$, $x^T y = 0$

$$\left(A^{-1}B\right)\left(A^{-1/2}\vec{x}\right) = A^{-1/2} A^{-1/2} B A^{-1/2} \vec{x}$$

$$= A^{-1/2} \lambda_1 \vec{x} = \lambda_1 A^{-1/2} \vec{x}$$

$$\left(A^{-1/2} y\right)^T A \left(A^{-1/2} x\right) = y^T x = 0$$

Conclusion: E-vecs of W are $\perp$ wrt $\langle , \rangle$