M408S Concept Inventory

These questions are open-ended, and are intended to cover the main topics that we learned in M408S. These are *not* crank-out-an-answer problems! (There are plenty of those in the book, and you need to practice them, too.)

My suggestion is to *write out* answers to these questions. Then compare to the sample answers that I have provided in a separate document.

1) If $f(x)$ is a function of one variable, what does $f'(a)$ mean? How do you compute it?

2) What is an anti-derivative?

3) What does the definite integral $\int_a^b f(x)dx$ mean?

4) How do we compute it?

5) What are some of the things that we can compute using definite integrals?

6) What are some of the strategies for finding anti-derivatives (which we usually call "integrals" out of laziness)? Under what circumstances do you use each one?

7) What does the differential equation $\frac{dy}{dx} = 3y$ tell you about y as a function of x? (Answer this WITHOUT solving the differential equation. What does the differential equation actually say?)

8) What sorts of differential equations do you know how to solve? How do you solve them?

9) What models of growth do you understand? When do you use exponential growth vs. logistic growth?

10) What does the statement $\lim_{n\to\infty} 2^{-n} = 2$ mean? Is the statement true?

11) What does the statement $\sum_{n=0}^{\infty} 2^{-n} = 2$ mean? Is the statement true?

12) What's the difference between a sequence and a series?

13) What does convergence mean for (a) improper integrals, (b) sequences, and (c) series?

14) What are some of the techniques for telling whether a series converges? When do you use each one?

15) Define the radius of convergence and the interval of convergence of a power series.

16) When expressing a (known) function as a power series, how do you find the coefficients?

17) How do you take the derivative or integral of a power series?

18) What are the Maclaurin series for e^x, $\sin(x)$, $\cos(x)$, $\ln(1+x)$, $\tan^{-1}(x)$ and $(1+x)^k$?

19) How accurate is the approximation $T_k(x) \approx f(x)$, where $T_k(x)$ is a k-th order Taylor polynomial.

20) What is Taylor series good for?

21) If $f(x, y)$ is a function of two variables, then what does $\partial f/\partial x$ mean? How do you compute it? (Ditto for $\partial f/\partial y$, but if you understand $\partial f/\partial x$ you probably understand $\partial f/\partial y$.)

22) What do f_{xy} and f_{yx} mean? How are they related?

23) What is a double integral?

24) What is an iterated integral? How is that different from a double integral? How do you use iterated integrals to compute double integrals?

25) What are Type-I and Type-II regions? How do you do a double integral over a Type-I region? Over a Type-II region?

26) What is switching the order of integation? How does that work?

27) What are some of the things that you can compute using double integrals?