M408S Homework 10. Due Monday, April 1

Part 1: Integral Test

1) Consider the function $f(x) = 1/x^2$. Let $s_n = \sum_{i=1}^{n} f(i)$. Let $I(n) = \int_1^n f(x)dx$. With the help of a calculator or computer, compute s_{10} and $I(10)$. How different are they? Then compute s_{100} and $I(100)$, and then s_{1000} and $I(1000)$. How is $s_n - I(n)$ behaving as a function of n? Can you make a prediction about what $s_{1,000,000} - I(1,000,000)$ is close to?

The point of Problem 1 is that the partial sum s_n and the integral $I(n)$ track together. If one of them gets big, then the other gets big. If one of them stays bounded, then so does the other. The improper integral $\int_1^\infty f(x)dx$ is the limit of $I(n)$ as $n \to \infty$. The infinite sum $\sum_{i=1}^{\infty} f(i)$ is the limit of s_n as $n \to \infty$. They behave the same, and that's the point of the integral test.

2) Now let $f(x) = 1/x$. As in problem 1, compute s_{10}, $I(10)$, s_{100}, $I(100)$, s_{1000}, and $I(1000)$, and make a table of n, s_n, $I(n)$, and $s_n - I(n)$. What do you estimate $s_{1,000,000} - I(1,000,000)$ will be?

3) With $f(x)$, etc. as in problem 2, how big does n have to be for s_n to be bigger than 5? Bigger than 10? Bigger than 15? (To solve this, program a calculator or computer to keep adding terms until you hit 5, or 10, or 15 and see how many steps it takes.) Do you see a pattern? Meanwhile, how big does n have to be for $I(n)$ to reach 5, or 10, or 15? (You should be able to do this very quickly with calculus and without a computer.)

4) With $f(x)$, etc as in problem 2, estimate how big n has to be for s_n to hit 30. How about 40? 100? 1000? (Do NOT do this by programming a calculator and letting it run. The sun would go nova, destroying the Earth, long before the calculation finished.)

5) If you actually try to add up the terms of $\sum 1/n$ on a computer, doing your arithmetic with 14 digit accuracy, then you'll stop growing after the 2×10^{14}th term, since after that $1/n$ will be zero to 14 decimal places and the computer won't be able to tell the difference between $a_n = 1/n$ and 0. Roughly how big will s_n get by then?

Part 2: Comparison Tests

Stewart, Section 11.4, Problems 10, 14, 20, 22. Also, does the series $\sum_{n=1}^{\infty} \sin(1/n^2)$ converge? Why or why not?