

M408S Homework 11. Due Monday, April 8

Part 1: Alternating series and absolute convergence

A lot of you said that rearrangements were confusing. The following problems are designed to help cut through the fog.

1) Consider the series $1 - 1 + (1/2) - (1/2) + (1/3) - (1/3) + (1/4) - (1/4) + \dots$. Show that this series converges. (To what?) Does it converge absolutely or conditionally?

2) Now rearrange the series so that we have 2 positive terms then a negative term, then two more positive terms, then a negative term, and so on. In other words, $1 + (1/2) - 1 + (1/3) + (1/4) - (1/2) + (1/5) + (1/6) - (1/3) + \dots$. Show that the rearranged series still converges, but to a different value. You should be able to figure out what this value is, based on last week's problems about the integral test and $\sum(1/n)$.

3) Now rearrange the series so that we have one positive term, then one negative, then 2 positive, then one negative, then 4 positive, then one negative, then 8 positive, then one negative, and so on. Show that this series diverges.

4) Now consider the series $1 - 1 + (1/2) - (1/2) + (1/4) - (1/4) + (1/8) - (1/8) + \dots$. Show that this converges absolutely. Then do rearrangements analogous to problems 2 and 3 and see what happens. Do the rearranged series still converge? If so, to what values? Why?

5) Stewart, page 731, problems 32 and 34.

Part 2: Root and ratio tests

Stewart pages 737-8, problems 4, 8, 14, 32.