M408S Homework 8. Due Monday, March 18

Part 1: Differential equations

1) (This problem is essentially problem 49 from Stewart Section 9.3) When a raidrop falls, it increases in size and so its mass at time t is a function of t, namely $m(t)$. The rate of growth of the mass is $km(t)$ for some positive constant k. Newton's law of gravity says that $(mv)' = mg$, where $v(t)$ is the downwards velocity of the raindrop at time t. (Note that Newton's law does NOT say that $v' = g$, since the mass is not constant. Absorbing stationary water molecules tends to slow the raindrop down, while gravity speeds it up.) (a) Turn this into a differential equation for v, given what you already know about m. (b) Solve this differential equation to get $v(t)$. (c) Compute $\lim_{t\to\infty} v(t)$ in terms of g and k. This limit is called the terminal velocity of the raindrop.

You are welcome to check your answers in the back of the book, but of course you still need to derive the answers and write them up yourself.

Stewart Section 9.3, problem 45

Stewart Section 9.4, problems 8 and 16.

Part 2: Sequences

6) Let a_n be the slope of the line from $(0, 1)$ to $(1/n, e^{1/n})$. Does the sequence $\{a_n\}$ converge? If so, to what value? What is the significance of the limit?

7) Let $a_n = \frac{1}{n^2}\sum_{k=1}^{n} k$. Does the sequence $\{a_n\}$ converge? If so, to what value? What is the significance of the limit? Is this something you have seen in calculus before?

8) Let $a_1 = 1$, and let $a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}$. Compute the first few terms of this sequence. Does is seem to be converging? If so, to what number? (Again, have you seen this before in calculus?)

Stewart, Section 11.1, problems 30, 38 and 68.