

## Volumes

State the volume  $V$  of the solid that lies between  $x = a$  and  $x = b$ . If the cross-sectional area of  $S$  in the plane  $P_x$ , through  $x$  and perpendicular to the  $x$ -axis, is  $A(x)$ , where  $A$  is a continuous function, then the volume  $V$  of  $S$  is

We like to find the volumes of solids generated by rotating a region about different axes. Explore the different methods below. Find the volume of the solids obtained by rotating the region bounded by the given curves about the specified lines. Sketch the scenarios!

1. About the  $x$ -axis. Our favorite!

$$y = 2 - \frac{1}{2}x, \quad y = 0, \quad x = 1, \quad x = 2; \text{ about the } x\text{-axis.}$$

2. About a horizontal axis.

$$x = y^2, \quad y = x^2; \text{ about the line } y = 1$$

3. About the  $y$ -axis

$$x = 2\sqrt{y}, \quad x = 0, y = 9; \text{ about the } y\text{-axis.}$$

4. About a vertical axis.

$$y = x^3, y = 0, x = 1; \text{ about } x = 2.$$

Sometimes we like to find the volumes of solids that are not obtained by rotating about an axis but instead have interesting cross sections. Find the volume of a pyramid with height  $h$  and base an equilateral triangle with side  $a$  (a tetrahedron). (See # 52). To help you, see also example 8.