M408S Final Exam, May 13, 2013

1. Areas and volumes. (8 pts) Let R be the region in the x-y plane between the x axis, the curve $y = 1/x$ and the line $x = 1$. Let V be the 3-dimensional solid obtained by rotating R around the x axis.

a) Express the area of R as an improper integral. Does this integral converge? If so, evaluate the integral. If not, explain why it diverges.

b) Express the volume of V as an improper integral. Does this integral converge? If so, evaluate the integral. If not, explain why it diverges.

2. Integrals and limits. (12 pts) Evaluate the following integrals and limits. If an improper integral or a limit does not exist, say "does not exist" or "diverges".

a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}}$.

b) $\displaystyle\lim_{x\to 0} \frac{\sin(x) - x\cos(x)}{x^3}$.

c) The sequence $\{a_n\}$ with $a_n = \dfrac{\sin(n)e^n}{ne^n+1}$.

3. (16 pts) For each of these series, indicate whether the series converges absolutely, converges conditionally, or diverges. Give a short explanation of why (e.g. "converges by comparison to $\sum 2^{-n}$").

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\pi n/2)}{n}$.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n\sin(1/n)}{n^2+1}$.

c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^n}{n!}$.

d) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2(-1)^n + e^{-n}}{n^2+4n+1}$.

4. (10 points) Taylor polynomials

a) Compute the second order Taylor polynomial $T_2(x)$ for $f(x) = \tan(x)$ around $a = \pi/4$.

b) Use this polynomial to approximate $\tan(\frac{\pi}{4} + 0.1)$.

5. Partial derivatives. (8 pts) Consider the function
$f(x,y) = \ln(2+x+y)e^{x-y} - \ln(2)$.

a) Compute the partial derivatives f_x and f_y.

b) Evaluate these partial derivatives at $(x, y) = (0, 0)$.

Bonus (4 pts): Use these partial derivatives to approximate $f(-0.01, 0.02)$. You can use the approximation $\ln(2) \approx 0.7$.

6. (16 pts) Double integrals. Let R be the region in the x-y plane bounded by the curve $y = \ln(x)$, the line $x = 1$ and the line $y = 2$.

a) Compute the area of R. (There are several ways to do this.)

b) Convert the double integral $\iint_R 2\pi x dA$ into an iterated integral where you integrate first over y and then over x. Be explicit with your limits of integration!

c) Convert the double integral $\iint_R 2\pi x dA$ into an iterated integral where you integrate first over x and then over y. As with the previous part, be explicit with your limits of integration.

d) Evaluate $\iint_R 2\pi x dA$ by whichever method you prefer.

There were also 6 Quest problems, each worth 5 points available as a separate file.