M382D Midterm Exam, March 11, 2016

Do **three** out of four problems. (The fourth is on the back.) Be clear about which problems you want graded. If you attempt all four problems, I'll just grade the first three.

1a) Show that CP^1 is a manifold by explicitly constructing a smooth atlas.

1b) The linear map $C^2 \to C^2$ given by the matrix $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ sends 1-dimensional subspaces of C^2 to 1 dimensional subspaces of C^2, and so induces a map $f : CP^1 \to CP^1$. Let $x \in CP^1$ be the line spanned by $(1, 1)$. Compute the matrix of df_x with respect to coordinates that you already defined in part (a).

2) Let X be a manifold (without boundary) of arbitrary dimension, and let $f : X \to \mathbb{R}^2$ be an arbitrary smooth map. Show that there exists a line $L \subset \mathbb{R}^2$ (not necessarily through the origin) such that $f \pitchfork L$.

3) Let K be the Klein bottle, obtained by identifying edges of the unit square as in the figure below. Let C be the subset $x = 1/2$, where (x, y) are the usual coordinates on the unit square. **Using intersection theory**, show that the inclusion $i_C : C \to K$ is not homotopic to a constant map. (There are lots of ways to do this using the machinery of algebraic topology, such as knowing that C is one of the standard generators of the fundamental group of K. Those do NOT count. Do this using the machinery we developed THIS semester.)

4) Let $X \subset \mathbb{R}^n$ be a compact manifold, $Y \subset \mathbb{R}^m$ a compact manifold, and Z_0 and Z_1 closed sub-manifolds of Y. Suppose that $f_0 \pitchfork Z_0$, that $f_1 \pitchfork Z_1$, and that f_0 and f_1 are homotopic. Show that there is a smooth map $G : [0,1] \times X \to Y$ with the following properties:

- The restriction of G to $(0,1) \times X$ is transversal to both Z_0 and Z_1.
- $g_0 = f_0$ (where $g_0(x) := G(0,x)$).
- $g_1 = f_1$.

(Note: we can't require that G itself be transversal to both Z_0 and Z_1 since this would involve properties of f_0 and f_1.)