M408M Final Exam, December 9, 2015

The exam is closed book, but you may have two hand-written 8.5×11 crib sheets. Calculators are **NOT** allowed. Neither are electronic devices of **ANY** kind. Please turn your cell phones, ipods, etc OFF and stow them away and out of reach.

Don't forget to **justify your answers!** Part credit will be given, but not if I can't figure out where you got your answer. Even correct answers may not get full credit if I can't understand your reasoning.

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Full Name and section time (8AM vs. 3PM)

Problem	Score
1	/12
2	/12
3	/10
4	/12
5	/10
6	/12
7	/12
8	/10
9	/12
Total	/102

1) A polar curve. (2 pages!) Let C be the portion of the "cloverleaf" curve $r = \sin(2\theta)$ that lies in the first quadrant.

a) Draw a rough sketch of C.

b) Write down an integral that gives the arc-length of C. Simplify the integrand as much as possible, but **do not attempt to compute the integral.** (It can't be done in closed form).

c) Compute the area enclosed by C. (This integral **can** be done in closed form, and I expect you to do it.)

2. Lines and planes. (2 pages!) Let $P(3,-1,4)$, $Q(2,1,7)$, and $R(1,5,8)$ be points in $\mathbb{R}^3$. Let $\mathcal{L}$ be the line through P and Q, and let $\mathcal{T}$ be the plane containing all three points.

a) Give a parametrization for $\mathcal{L}$.

b) Write down the symmetric equations for $\mathcal{L}$. (That is, the equations relating x, y and z that don't involve the parameter t.)

c) Find a vector normal to $\mathcal{T}$.

d) Find the equation of $\mathcal{T}$. Simplify as much as possible.

3. Curves. Consider the curve $\mathbf{r}(t) = (1 + t^2, 3 - 2\ln(t), 5 + 2\sqrt{2}(t - 1))$.

a) Find the arc-length of the curve traced out as t goes from 1 to 3.

b)When $t = 1$, this curve goes through the point $P(2, 3, 5)$. Find the tangent, principal normal, and binormal vectors at this point.

4. Consider the function $f(x, y) = xy^3 - x^2y$.

a) Compute the partial derivatives f_x and f_y (as functions of x and y).

b) The surface $z = f(x, y)$ contains the point $(-3, 2, -42)$. Find the equation of the plane tangent to the surface at this point.

c) Using linearization, differentials, or the answer to (b) (all of which amount to essentially the same thing), approximate $f(-2.999, 3.002)$.

5. Level surfaces. Consider the surface $e^x + 2y + y\ln(z) = 7$. This goes through the point (0,3,1).

a) Find a vector normal to the surface at the point (0,3,1).

b) Find the equation of the plane tangent to the surface at that point.

6. Max/min. Consider the function $f(x, y) = e^y(y^2 - x^2)$.

a) Compute the partial derivatives f_x, f_y, f_{xx}, f_{xy} and f_{yy}.

b) Find all the critical points of this function.

c) For each critical point, use the second derivative test to determine whether the critical point is a local maximum, a local minimum, or a saddle point.

7. Double integrals in Cartesian coordinates. (2 pages!)

a) Compute $\iint_{D_a} \frac{3x}{y} dA$ where D_a is the region bounded by the lines $x = 0$, $x = 1$, and $y = 1$ and the curve $y = 2e^x$.

b) Compute $\iint_{D_b} x^2 y dA$ where D_b is the region bounded by the lines $y = 0$, $y = 1$, and $x = 0$ and the curve $y = \ln(x)$.

c) Rewrite the iterated integral $\int_1^3 \int_{x^2}^{4x-3} \cos(x^2y^3)dy\,dx$ as an iterated integral $dxdy$. (That is, swap the order of integration.) You do **NOT** need to evaluate the resulting iterated integral!

8. Laminae. Suppose we have a fan blade in the shape of the region you considered in problem 1. That is, it is bounded by the polar curve $r = \sin(2\theta)$ in the first quadrant. [Note that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.] The density of this blade is given by $\rho(x, y) = \frac{x}{x^2+y^2}$. [Yes, the density blows up as we approach the origin, but all of the integrals in this problem still converge. Also, this problem is best done in polar coordinates.]

a) Compute the mass of the blade.

b) Compute the moment of inertia I_0. [The last step in the integral is a little tricky. Remember that $\sin^2(\theta) + \cos^2(\theta) = 1$.]

9. Mappings. (2 pages!) Let D be the parallelogram (actually a square) whose corners are $(0,0)$, $(3,-1)$, $(1,3)$ and $(4,2)$. Our goal is to compute $\iint_D e^{(x+y)/2} dA$.

a) Find a mapping that sends the unit square to D.

b) Rewrite our integral as an integral over the unit square. Don't forget the Jacobian!

c) Evaluate that new-and-improved integral.