M408D First Midterm Exam, February 19, 2016

1) (30 pts) Strategy for integration. Indicate the key technique for evaluating the following integrals. **You do not need to actually finish the integrals**, but you have to provide a clear road map for **how** to compute them. For instance, you answer might be "u-substitution with $u = e^x$" or "integration by parts with $u = x^2$, $dv = e^x dx$", or "integrate by parts twice to go in circles" or "trig substitution with $x = 2\sin(\theta)$, or "partial fractions with $\frac{A}{x} + \frac{Bx+C}{x^2+7}$" or "reduce degree with double-angle formula", etc. Just naming a technique like "u-sub" is only worth part credit. Describing how it applies is worth full credit.

a) $\int \dfrac{4dx}{9 + (x-2)^2}$

b) $\int \dfrac{4dx}{x^3 - x^2}$

c) $\int (x^2 + 5)\ln(x)dx$.

2. (30 points) Compute the following quantities:

a) $\int_0^1 (x^2 + 3)^2 dx$.

b) $\int_0^{\pi/2} \sin^3(x)dx$

c) $\sin(\tan^{-1}(5/3))$.

3. (20 pts) Differential equations

a) Find the general solution to the differential equation $\frac{dy}{dx} = e^x y^2$.

b) Find the solution to $\dfrac{dy}{dx} = \dfrac{5y}{x} + 8$ with initial condition $y(1) = 7$.

4. (2 pages, 20 points) A cold beer is heating up at a rate proportional to the difference between room temperature (75 degrees Farenheit) and the temperature of the beer.

a) Write down a differential equation for the function $T(t)$ that describes the data I just gave you. Here $T(t)$ is the temperature of the beer, in degrees Farenheit, and t is the number of minutes since pouring. (Your answer may involve some unknown proportionality constant.)

b) Find the general solution to this equation. This may involve an unknown proportionality constant and/or an arbitrary constant of integration.

c) Now suppose that when the beer is poured, it is at 35 degrees. 10 minutes later, it is at 55 degrees. Use this information to figure out the unknown constant(s) and to determine the temperature of the beer as a function of time.

d) At what time will the temperature of the beer be 70 degrees?