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#2 Negations of "All dogs are loyal"

c. Some dogs are disloyal.
f. There is a dog that is disloyal.

#3: a) $\exists$ fish $x$ such that $x$ does not have gills.

b) $\exists$ computer $c$ such that $c$ does not have a CPU.

c) $\exists$ movies $m$, $m$ is not over 6 hours long.

d) $\exists$ band $b$, $b$ has not won at least 10 Grammy Awards.

#9) The negation of the statement

"$\forall$ real numbers $x$, if $x > 3$, then $x^2 > 9$"

is the statement

"$\exists$ real number $x$ such that $x > 3$ and $x^2 \leq 9$."

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#13  The proposed negation is not a correct negation.

A correct negation is as follows:

There exists an integer n such that \( n^2 \) is even, but n is not even.

#17  The Negation:

\( \exists \) integer d such that \( \frac{6}{d} \) is an integer and \( d \neq 3 \).

#22  The Negation:

There is an integer which is not odd, but its square is odd.

Also OK: There is an integer whose square is odd, but it is not odd itself.

#34  Original Statement:

\( \forall \) animals \( x \), if \( x \) is a dog, then \( x \) has paws and \( x \) has a tail.

Contrapositive:

\( \forall \) animals \( x \), if \( x \) does not have paws or \( x \) does not have a tail, then \( x \) is not a dog.
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#32  ORIGINAL STATEMENT:
"If the square of an integer is odd, then the integer is odd."

This is True.

The Converse:
"If an integer is odd, then its square is odd."

This is True.

The Inverse:
"If the square of an integer is not odd, then the integer is not odd."

This is True.

The Contrapositive:
"If an integer is not odd, then its square is not odd."

This is True.

#412 If one obtains a Master's degree, then they passed a comprehensive exam.
The statement is the negation of a conditional:

It is not the case that having a large income is a necessary condition for a person to be happy.

Stated equivalently:

It is not the case that, if one does not have a large income, then one is not a happy person. ≡ If you are happy, then you have a large income.

Since the negation of a conditional statement is an AND statement: [\sim(p \rightarrow q) \equiv p \land \sim q]

This can be stated equivalently as:

There exists a person who is happy and does not have a large income.

Informally stated:

Some people without a large income are happy.

This is equivalent to the original statement because

\[ \sim(\forall x, \text{if } P(x) \text{ then } Q(x)) \equiv \exists x \text{ such that } P(x) \land \sim Q(x) \]
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#46: The statement is the negation of a conditional:

It is not the case that, if a function is a polynomial function, then it has a root which is a real number.

Because \( \neg (\forall x, \text{if } P(x), \text{then } Q(x)) \)

\[\equiv\]

\( \exists x \text{ such that } P(x) \text{ and } \neg Q(x) \)

This can be stated equivalently as:

There exists a function \( f \) such that \( f \) is a polynomial function and \( f \) does not have a root which is a real number.

Informally: Some polynomial functions do not have a real root.