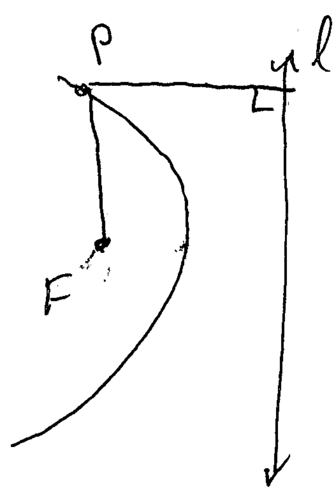
HW #3 is due Fri, 2/13

THE LECTURE ON POLAR EQUATIONS OF CONIC SECTIONS

For Monday, Read Sec's 12.1 and 12.2

Conics in Polar Coord's

Let F be a fixed point (Focus)
Let l be a fixed line (Directrix)
Let e be a positive real number, $e > 0$ (eccentricity)



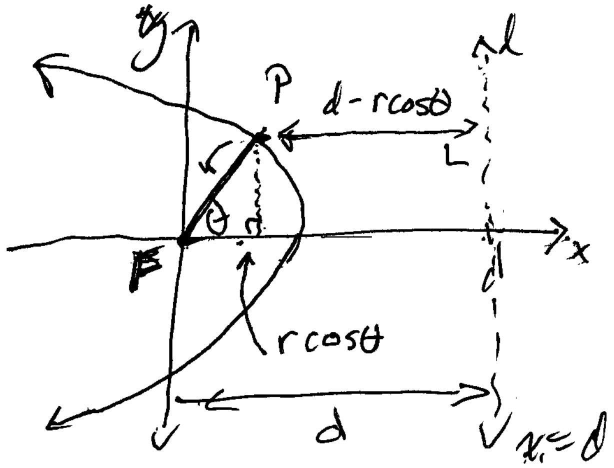
A conic section is the set of all points P such that $\frac{|PF|}{|Pl|} = e$

For e:	$\Rightarrow$	The Conic is:
$e < 1$		Ellipse
$e = 1$		Parabola
$e > 1$		Hyperbola.

For Polar Conics, place the Focus F at the Pole (Origin)

Suppose (for now) that the directrix is a vertical line with rectangular equation $x = d$, $d > 0$, and eccentricity $e \geq 0$.

$P = (r, \theta)$



$$\frac{|PF|}{|Pl|} = e = \frac{r}{d - r\cos\theta}$$

$$r = e(d - r\cos\theta)$$

$$r = ed - er\cos\theta$$

$$\Rightarrow r + er\cos\theta = ed \Rightarrow r(1 + e\cos\theta) = ed$$

$$\boxed{r = \frac{ed}{1 + e\cos\theta}}$$

← The polar Equation of the conic with vertical Directrix with equation $x = +d$

Note: The Graph always opens-out away from the directrix

In fact, with $e > 0$ and $d \neq 0$, A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \quad \text{OR} \quad r = \frac{ed}{1 \pm e\sin\theta}$$

is the equation of a conic with a focus at the pole.

Ex: Determine the vertex (or vertices) and sketch the graph of the polar conic

$$r = \frac{15}{3 - 2\cos\theta} \times \frac{1/3}{1/3}$$

$r = \dfrac{ed}{1 \pm e\cos\theta}$ Get "1" here

Sol'n: $r = \dfrac{5}{1 - \frac{2}{3}\cos\theta} \Rightarrow$ $e = \frac{2}{3}$, $ed = 5 = (\frac{2}{3})d$, $d = \frac{3}{2} \times 5 = \frac{15}{2} = 7.5$

Directrix is Vertical and (Red) Equation is $x = -d$, "$x = -7.5$" is the Directrix

$e = \frac{2}{3} < 1 \Rightarrow$ Ellipse!

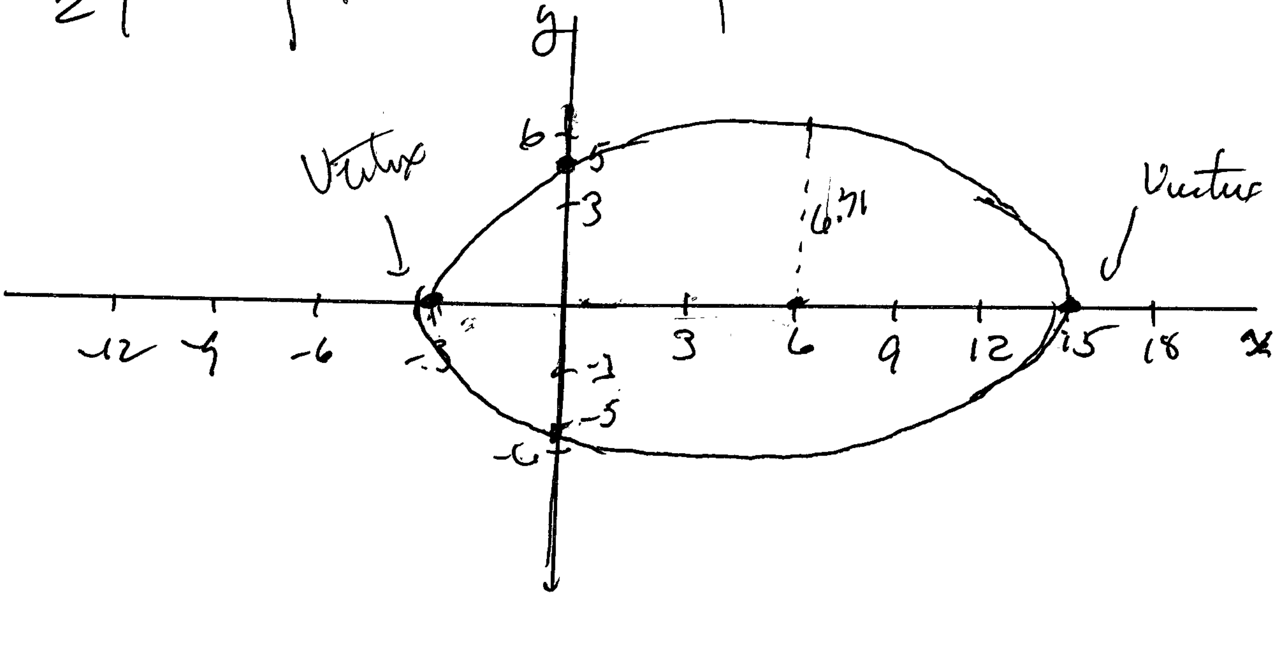
θ	$\cos\theta$	$r = \dfrac{15}{3 - 2\cos\theta}$	$(r,\theta)_P$
0	1	$r = 15/1 = 15$	$(15, 0)_P$
$\frac{\pi}{2}$	0	$r = 5 = 15/3$	$(5, \frac{\pi}{2})$
π	-1	$r = \frac{15}{5} = 3$	$(3, \pi)$
$\frac{3\pi}{2}$	0	$r = 5$	$(5, \frac{3\pi}{2})$

$2a = 15 - (-3) = 18$

$a = 9$

$15 - 9 = 6$

Center $(6, 0)$



FACT For Ellipses and Hyperbolas:

$$e = \frac{c}{a} \quad \text{(from } a, b, c \text{ in Rectangular Terms)}$$

Here $a = 9$, $e = \frac{2}{3} \Rightarrow \frac{2}{3} = \frac{c}{9} \Rightarrow c = 6$

Recall, $a^2 = b^2 + c^2 \Rightarrow$ $\cancel{c^2}$ $b^2 = a^2 - c^2 = 81 - 36$

$b^2 = 45$, $b = \sqrt{45} \approx 6.71$

Rect Eq:

(Before Shift)

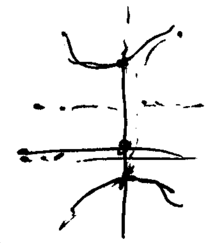
$$\frac{x^2}{81} + \frac{y^2}{45} = 1,$$

$(0,0) \to (6,0)$ After Shift

$$\boxed{\frac{(x-6)^2}{81} + \frac{y^2}{45} = 1}$$

Ex: Sketch the Conic

$$r = \frac{32}{3+5\sin\theta} \times \frac{1/3}{1/3}$$



$\sin\theta \longrightarrow$ Horizontal directrix

$$r = \frac{32/3}{1+\frac{5}{3}\sin\theta}$$

$e = \frac{5}{3} > 1 \Rightarrow$ Hyperbola!

$ed = \frac{32}{3} = \left(\frac{5}{3}\right)d$ $\quad r = \frac{ed}{1+e\sin\theta}$

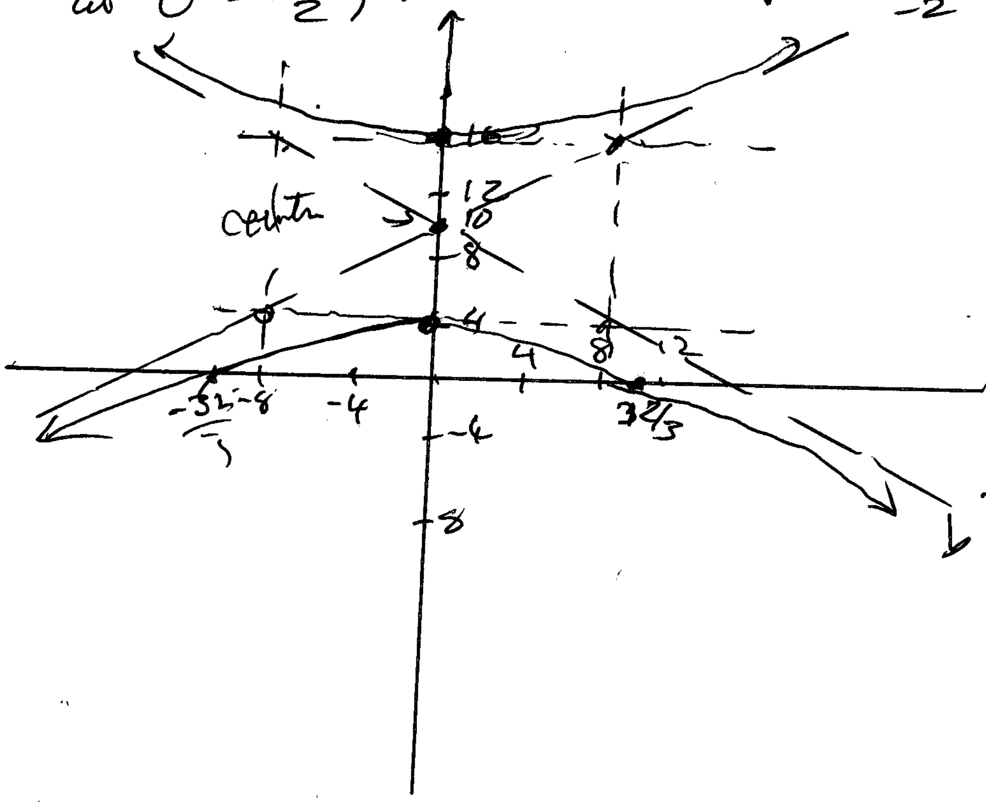
$d = \frac{32}{5} = 6.4$

Directrix : $y = +6.4$

at $\theta = 0$ and $\theta = \pi$, $\sin\theta = 0$, $r = \frac{32}{3} = 10.66\ldots$

at $\theta = \frac{\pi}{2}$, $\sin\theta = 1 \Rightarrow r = \frac{32}{8} = 4$ $\quad (4, \frac{\pi}{2})$

at $\theta = \frac{3\pi}{2}$, $\sin\theta = -1 \Rightarrow r = \frac{32}{-2} = -16$ $\quad (-16, \frac{3\pi}{2})$ $\Big\}$ Vertices



~~Center~~

$2a = 16 - 4 = 12$

$a = 6$

$e = \frac{5}{3} = \frac{c}{a} = \frac{c}{6} \Rightarrow c = 10$

$b^2 = c^2 - a^2 \Rightarrow b^2 = 100 - 36 = 64$

$b^2 = 64$

$b = 8$