~~HW #9 is due Fri.~~

TOPICS:-
IVP's, Solution Curves

Differential EQUATIONS

$y = f(x)$:

<u>Ex:</u> $y'' + 4y = 0$

A <u>particular solution</u> is $y = 7\sin 2x$.

$y' = (7 \cdot \cos 2x) \cdot 2 = 14\cos 2x$

$y'' = -28\sin 2x$; $4y = 28\sin 2x$

$y'' + 4y = (-28\sin 2x) + (28\sin 2x) = 0$

The <u>General Solution</u> of this D.E. is

$y = C_1 \cos 2x + C_2 \sin 2x$ where

C_1 and C_2 any real numbers.

When $C_1 = 0$ and $C_2 = 7$, this produces

$y = 0\cos 2x + 7\sin 2x = \underline{7\sin 2x}$

An Initial Value Problem (IVP) consists of a differential equation And particular ~~specified~~ specified values that the solution of interest must have. The specified values are called "Initial Conditions".

Ex of an IVP: Find a solution of the D.E $y'' + 4y = 0$ such that

$$y\left(\tfrac{\pi}{4}\right) = 3 \text{ and } y'\left(\tfrac{\pi}{4}\right) = 10.$$

Sol'n: Recall the General solution:

$$y = C_1 \cos(2x) + C_2 \sin(2x).$$

Set $x = \pi/4$, then solve for C_1 and C_2:

$$y(\pi/4) = C_1 \cos(\pi/2) + C_2 \sin(\pi/2) = 3$$

$$= 0 + C_2 \cdot 1 = 3 \Rightarrow \underline{C_2 = 3}$$

$$y'(x) = -2C_1 \sin(2x) + 2C_2 \cos(2x)$$

$$y'(\pi/4) = -2C_1 \cdot \sin(\pi/2) + 2 \times 3 \cos(\pi/2) = 10$$

$$= -2C_1 \cdot 1 + 6 \cdot 0 = 10 \Rightarrow$$

$$-2C_1 = 10$$

$$\Rightarrow \underline{C_1 = -5}$$

The Solution of this IVP is

$$\boxed{y = -5\cos(2x) + 3\sin(2x)}$$

Consider the D.E. $4yy' - 2x = 0$.

The general solution y is only implicitly defined:

$$2y^2 - x^2 = C \qquad -\infty < C < \infty.$$

Check: $2(2yy') - 2x = 0$
$4yy' - 2x = 0$

The graph of a particular solution of a D.E. is called a solution curve of the D.E.

The "Solution Curves of a D.E." consists of all the graphs of the general solution.

For the D.E $4yy' - 2x = 0$,

The general solution is $2y^2 - x^2 = C$

Draw "the solution curves" of this D.E.

For $C = 0$

The curve is

$$2y^2 - x^2 = 0$$

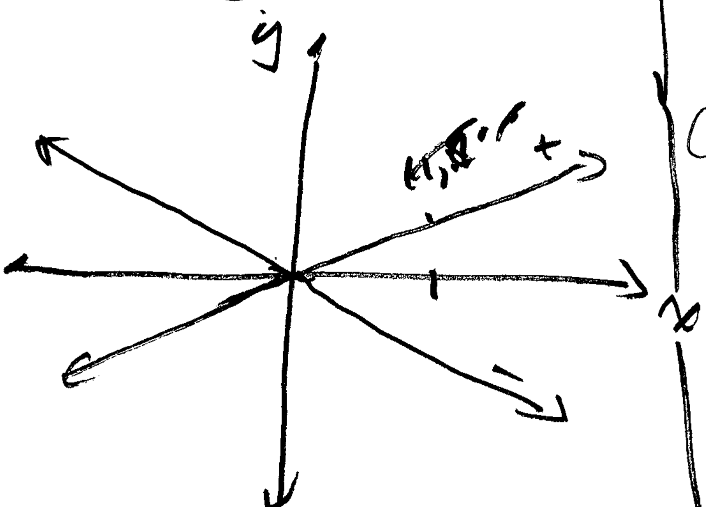
$$2y^2 = x^2$$

$$\sqrt{2}\,|y| = |x|$$

$$|y| = \frac{1}{\sqrt{2}}|x|$$

$$y = \pm\frac{1}{\sqrt{2}}|x|$$

$$y \doteq \pm .7071\,|x|$$

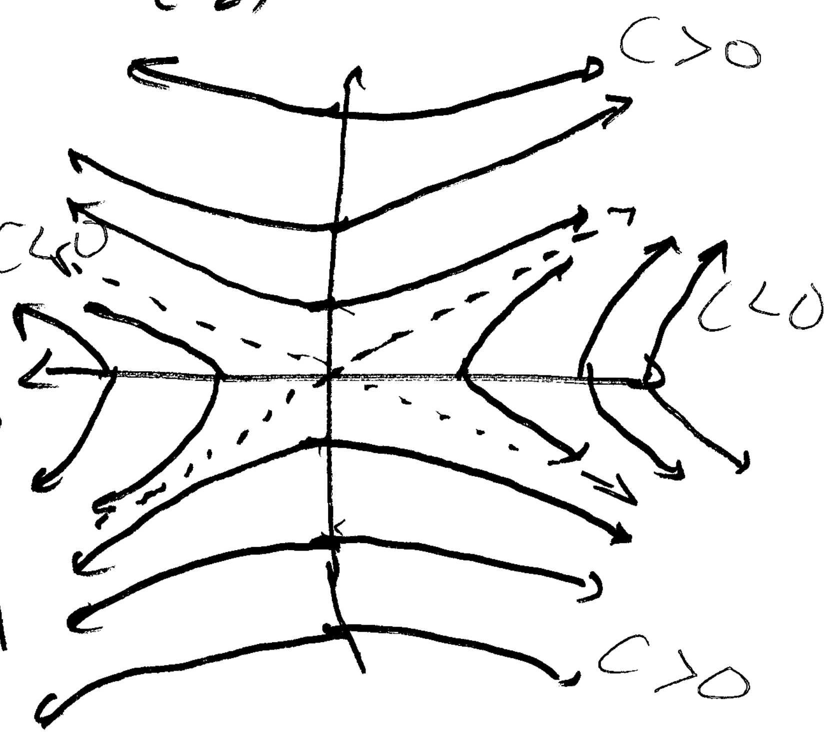


For $C \neq 0$

$$2y^2 - x^2 = C$$

$$\frac{2y^2}{C} - \frac{x^2}{C} = 1$$

$$\therefore \frac{y^2}{(C/2)} - \frac{x^2}{C} = 1$$



Ex: Let k be a real number.

Consider the D.E. $\dfrac{dP}{dt} = kP$, $P = f(t)$.

One solution of this D.E. is $P(t) = 0$ for all real # t.

Assume $P \neq 0$. Then, Divide by P:

$$\frac{P'}{P} = \frac{dP/dt}{P} = k$$

$$\Rightarrow \int \frac{P'(t)}{P(t)}\,dt = \int k\,dt$$

$$\ln|P(t)| = kt + C \quad \text{for } C \text{, a constant.}$$

$$e^{\ln(|P(t)|)} = e^{(kt+c)}$$

$$|P(t)| = e^{kt} \cdot e^{c}$$

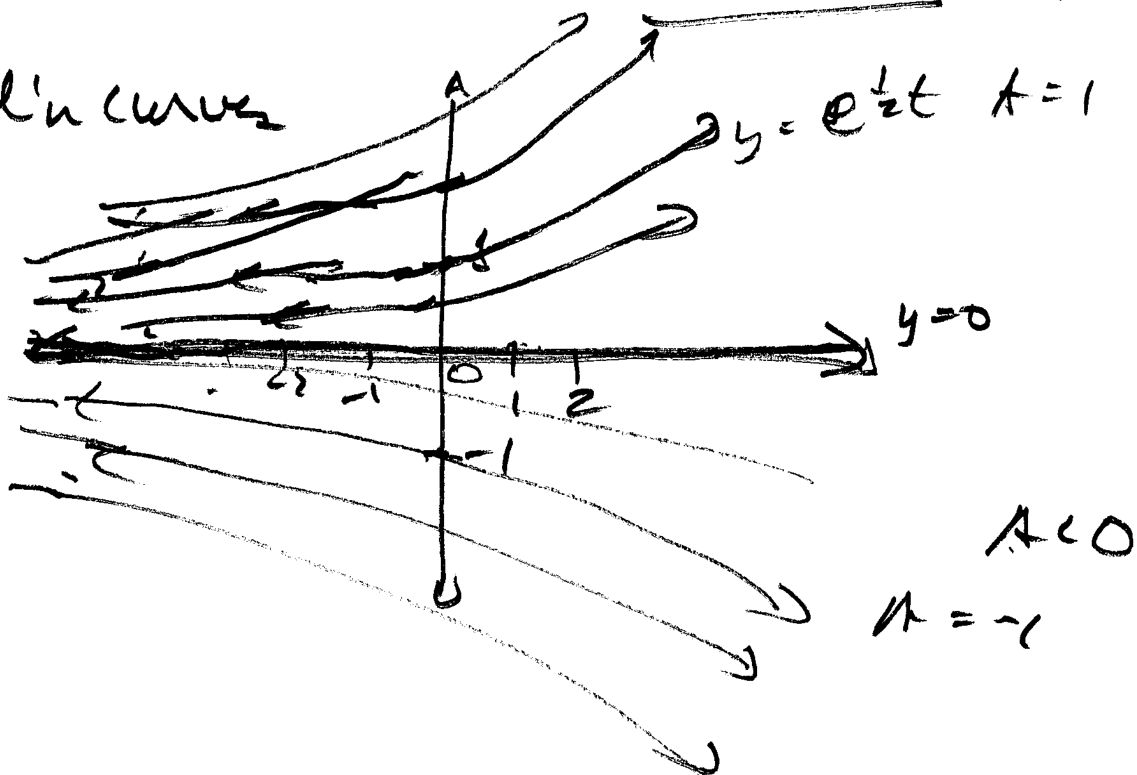
$P(t) = \pm A e^{kt}$ where A is any positive real #.

$\boxed{P(t) = Ae^{kt} \text{ where } A \text{ is any real \#.}}$

The General Solution of $P' = kP$.

For $k = \frac{1}{2}$, the D.E. $P' = \frac{1}{2}P$,
and its general sol'n is $P(t) = Ae^{\frac{1}{2}t}$. $A>0$

The sol'n curves



Ex: of Another IVP Problem

For $P'(t) = \frac{1}{2}P$,

Solve $P' = \frac{1}{2}P$ and $P(0) = \frac{1}{3}$

Sol'n: General sol'n: $P(t) = Ae^{\frac{1}{2}t}$

at $t=0$, $P(0) = Ae^0 = A = \frac{1}{3}$

$$\boxed{P(t) = \frac{1}{3}e^{\frac{1}{2}t}}$$ ← The sol'n to the IVP.

The D.E. $(y')^2 = -1$ has no sol'n (7)