A Separable First-Order Differential Equation Application

A full 200-gallon tank has brine (salt water) with 25 lbs of salt in the tank. Starting at time $t = 0$ min, a brine solution with concentration 0.05 lbs / gal enters the tank at the constant rate of 10 gal/min, and the well-stirred solution is drained from the tank at the same rate, 10 gal/min.

(a) Determine the amount of salt in the tank as a function of time **t**.

(b) When does the amount of salt in the tank become 15 lbs ?

Solution: Let $y(t)$ = # of lbs of salt in the tank at time t minutes. The Initial Condition is $y(0) = 25$.

We Discover a Differential EQUATION in function y which has the function $y(t)$ as a solution.

$$\frac{dy}{dt} = \text{RATE}_{\text{IN}} - \text{RATE}_{\text{OUT}} \quad \text{in terms of } \frac{\text{lbs of SALT}}{\text{MIN}}.$$

BRINE CONCENTRATION × RATE OF FLOW

$$\text{RATE IN} = (0.05 \text{ lbs/gal}) \times (10 \text{ gal/min})$$
$$= 0.5 \text{ lbs of salt / min}.$$

BRINE CONCENTRATION × RATE OF FLOW

$$\text{RATE}_{\text{OUT}} = \left(\frac{y(t) \text{ lbs}}{200 \text{ gal}}\right) \times (10 \text{ gal/min})$$
$$= \frac{y}{20} \text{ lbs of salt / min}$$

$$\frac{dy}{dt} = 0.5 - \frac{y}{20} = \frac{10}{20} - \frac{y}{20} = \frac{1}{20}(10 - y)$$

The Initial Condition is $y(0) = 25$.