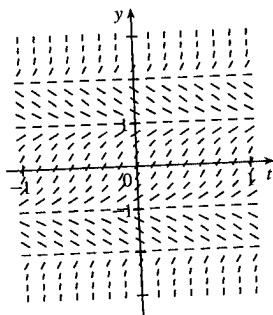


18.



Note that when $f(y) = 0$ on the graph in the text, we have $y' = f(y) = 0$; so we get horizontal segments at $y = \pm 1, \pm 2$. We get segments with negative slopes only for $1 < |y| < 2$. All other segments have positive slope. For the limiting behavior of solutions:

- If $y(0) > 2$, then $\lim_{t \rightarrow \infty} y = \infty$ and $\lim_{t \rightarrow -\infty} y = 2$.
- If $1 < y(0) < 2$, then $\lim_{t \rightarrow \infty} y = 1$ and $\lim_{t \rightarrow -\infty} y = 2$.
- If $-1 < y(0) < 1$, then $\lim_{t \rightarrow \infty} y = 1$ and $\lim_{t \rightarrow -\infty} y = -1$.
- If $-2 < y(0) < -1$, then $\lim_{t \rightarrow \infty} y = -2$ and $\lim_{t \rightarrow -\infty} y = -1$.
- If $y < -2$, then $\lim_{t \rightarrow \infty} y = -2$ and $\lim_{t \rightarrow -\infty} y = -\infty$.

19. (a) $y' = F(x, y) = y$ and $y(0) = 1 \Rightarrow x_0 = 0, y_0 = 1$.

(i) $h = 0.4$ and $y_1 = y_0 + hF(x_0, y_0) \Rightarrow y_1 = 1 + 0.4 \cdot 1 = 1.4$. $x_1 = x_0 + h = 0 + 0.4 = 0.4$,
so $y_1 = y(0.4) = 1.4$.

(ii) $h = 0.2 \Rightarrow x_1 = 0.2$ and $x_2 = 0.4$, so we need to find y_2 .

$$y_1 = y_0 + hF(x_0, y_0) = 1 + 0.2y_0 = 1 + 0.2 \cdot 1 = 1.2,$$

$$y_2 = y_1 + hF(x_1, y_1) = 1.2 + 0.2y_1 = 1.2 + 0.2 \cdot 1.2 = 1.44.$$

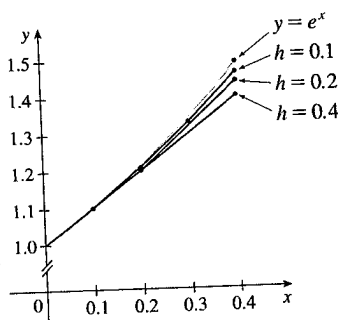
(iii) $h = 0.1 \Rightarrow x_4 = 0.4$, so we need to find y_4 . $y_1 = y_0 + hF(x_0, y_0) = 1 + 0.1y_0 = 1 + 0.1 \cdot 1 = 1.1$,

$$y_2 = y_1 + hF(x_1, y_1) = 1.1 + 0.1y_1 = 1.1 + 0.1 \cdot 1.1 = 1.21,$$

$$y_3 = y_2 + hF(x_2, y_2) = 1.21 + 0.1y_2 = 1.21 + 0.1 \cdot 1.21 = 1.331,$$

$$y_4 = y_3 + hF(x_3, y_3) = 1.331 + 0.1y_3 = 1.331 + 0.1 \cdot 1.331 = 1.4641.$$

(b)



We see that the estimates are underestimates since they are all below the graph of $y = e^x$.

- (c) (i) For $h = 0.4$: (exact value) - (approximate value) = $e^{0.4} - 1.4 \approx 0.0918$
(ii) For $h = 0.2$: (exact value) - (approximate value) = $e^{0.4} - 1.44 \approx 0.0518$
(iii) For $h = 0.1$: (exact value) - (approximate value) = $e^{0.4} - 1.4641 \approx 0.0277$

Each time the step size is halved, the error estimate also appears to be halved (approximately).