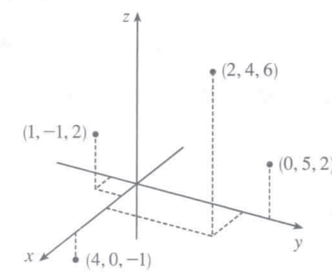
VECTORS AND THE GEOMETRY OF SPACE □ ET 12

mensional Coordinate Systems ET 12.1

origin, which has coordinates $(0, 0, 0)$. First we move
he positive x-axis, affecting only the x-coordinate,
the point $(4, 0, 0)$. We then move 3 units straight
the negative z-direction. Thus only the z-coordinate is
we arrive at $(4, 0, -3)$.

2.

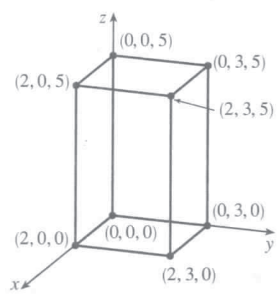


rom a point to the xz-plane is the absolute value of the y-coordinate of the point. $Q(-5, -1, 4)$ has the
vith the smallest absolute value, so Q is the point closest to the xz-plane. $R(0, 3, 8)$ must lie in the yz-plane
nce from R to the yz-plane, given by the x-coordinate of R, is 0.

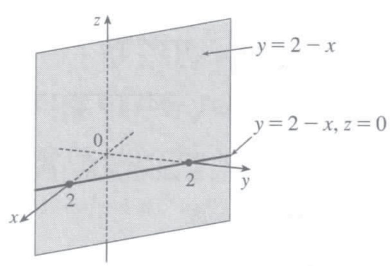
of $(2, 3, 5)$ on the xy-plane is $(2, 3, 0)$;
e, $(0, 3, 5)$; on the xz-plane, $(2, 0, 5)$.

the diagonal of the box is the distance between
$(2, 3, 5)$, given by

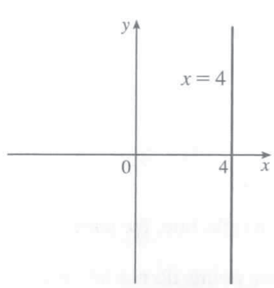
$$^2 + (3-0)^2 + (5-0)^2 = \sqrt{38} \approx 6.16$$

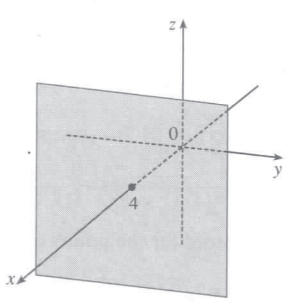


$x + y = 2$ represents the set of all points in
nd y-coordinates have a sum of 2, or
here $y = 2 - x$. This is the set
$\mid x \in \mathbb{R}, z \in \mathbb{R}\}$ which is a vertical plane
the xy-plane in the line $y = 2 - x$, $z = 0$.

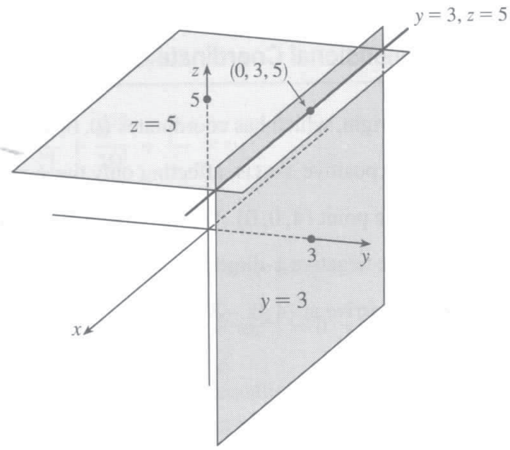


equation $x = 4$ represents a line parallel to
In $\mathbb{R}^3$, the equation $x = 4$ represents the
$z) \mid x = 4\}$, the set of all points whose
te is 4. This is the vertical plane that is
the yz-plane and 4 units in front of it.





(b) In $\mathbb{R}^3$, the equation $y = 3$ represents a vertical plane that is parallel to the xz-plane and 3 units to the right of it. The equation $z = 5$ represents a horizontal plane parallel to the xy-plane and 5 units above it. The pair of equations $y = 3$, $z = 5$ represents the set of points that are simultaneously on both planes, or in other words, the line of intersection of the planes $y = 3$, $z = 5$. This line can also be described as the set $\{(x, 3, 5) \mid x \in \mathbb{R}\}$, which is the set of all points in $\mathbb{R}^3$ whose x-coordinate may vary but whose y- and z-coordinates are fixed at 3 and 5, respectively. Thus the line is parallel to the x-axis and intersects the yz-plane in the point $(0, 3, 5)$.



7. We can find the lengths of the sides of the triangle by using the distance formula between pairs of vertices:

$$|PQ| = \sqrt{(7-3)^2 + [0-(-2)]^2 + [1-(-3)]^2} = \sqrt{16+4+16} = 6$$
$$|QR| = \sqrt{(1-7)^2 + (2-0)^2 + (1-1)^2} = \sqrt{36+4+0} = \sqrt{40} = 2\sqrt{10}$$
$$|RP| = \sqrt{(3-1)^2 + (-2-2)^2 + (-3-1)^2} = \sqrt{4+16+16} = 6$$

The longest side is QR, but the Pythagorean Theorem is not satisfied: $|PQ|^2 + |RP|^2 \neq |QR|^2$. Thus PQR is not a right triangle. PQR is isosceles, as two sides have the same length.

8. Compute the lengths of the sides of the triangle by using the distance formula between pairs of vertices:

$$|PQ| = \sqrt{(4-2)^2 + [1-(-1)]^2 + (1-0)^2} = \sqrt{4+4+1} = 3$$
$$|QR| = \sqrt{(4-4)^2 + (-5-1)^2 + (4-1)^2} = \sqrt{0+36+9} = \sqrt{45} = 3\sqrt{5}$$
$$|RP| = \sqrt{(2-4)^2 + [-1-(-5)]^2 + (0-4)^2} = \sqrt{4+16+16} = 6$$

Since the Pythagorean Theorem is satisfied by $|PQ|^2 + |RP|^2 = |QR|^2$, PQR is a right triangle. PQR is not isosceles, as no two sides have the same length.

9. (a) First we find the distances between points:

$$|AB| = \sqrt{(3-2)^2 + (7-4)^2 + (-2-2)^2} = \sqrt{26}$$
$$|BC| = \sqrt{(1-3)^2 + (3-7)^2 + [3-(-2)]^2} = \sqrt{45} = 3\sqrt{5}$$
$$|AC| = \sqrt{(1-2)^2 + (3-4)^2 + (3-2)^2} = \sqrt{3}$$

In order for the points to lie on a straight line, the sum of the two shortest distances must be equal to the longest distance. Since $\sqrt{26} + \sqrt{3} \neq 3\sqrt{5}$, the three points do not lie on a straight line.