COMPLETION AND CORRECTION OF WEDNESDAY'S LECTURE

There are 3 sections to this document.

<u>Part I</u>: The completion of the last example from the lecture, showing how to arrive at the value of the double integral:

$$\int_0^{2\pi}\int_0^3 \left(\sqrt{4r^2+1}\right) r\, dr\, d\theta = \frac{\pi}{6}\left(37\sqrt{37}-1\right).$$

<u>Part II</u>: An explanation and correction of the error I made in class when I tried to evaluate a double integral over a Type II polar region using polar coordinates.

<u>Part III</u>: The completion of the first example of finding surface area, where I will show how to arrive at the double integral:

$$A(S) = \iint_D \sqrt{(2x)^2+2^2+1}\, dA = \int_0^1\int_0^x (4x^2+5)^{\frac{1}{2}}\, dy\, dx$$

$$\ldots = \frac{1}{12}\left(27-5\sqrt{5}\right) \text{ square units.}$$

Part I:

$$\int_0^{2\pi}\int_0^3 \left(\sqrt{4r^2+1}\right) r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^3 (4r^2+1)^{\frac{1}{2}}\, r\,dr\,d\theta = I$$

use "$u = 4r^2+1$" substitution, in which
$du = 8r\,dr$, so $r\,dr = \frac{1}{8}du$.
When $r=0$, $u = (4\times 0^2+1) = 1$.
When $r=3$, $u = 4\times 3^2+1 = 37$.

$$I = \int_0^{2\pi}\frac{1}{8}\int_1^{37} u^{1/2}\,du\,d\theta = \int_0^{2\pi}\frac{1}{8}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^{37} d\theta$$

$$= \int_0^{2\pi}\left(\frac{1}{12}u\sqrt{u}\right)\Big|_1^{37} d\theta$$

$$= \int_0^{2\pi}\left(\frac{1}{12}\times 37\sqrt{37} - \frac{1}{12}\times 1\right) d\theta$$

$$= \int_0^{2\pi}\left(\frac{1}{12}(37\sqrt{37}-1)\right) d\theta = \frac{1}{12}(37\sqrt{37}-1)\int_0^{2\pi} 1\,d\theta$$

$$= \frac{1}{12}(37\sqrt{37}-1)\times(2\pi - 0)$$

$$= \frac{\pi}{6}(37\sqrt{37}-1)$$

Part II: When I look back at my lecture notes, I see that my mistake in class was that I copied the function $z = f(x,y)$ to be integrated over the petal incorrectly.

My notes said that the problem was to integrate $z = f(x,y) = \sqrt{x^2+y^2}$, but in class, I mistakenly said that the problem was to integrate

$z = f(x,y) = x^2+y^2$, a much more difficult problem to solve.

In this document, I will pose the correctly as I had intended to in class and then I will write the solution for you see.

Problem: Region D is the region inside the petal of a four-leaved rose, with polar equation

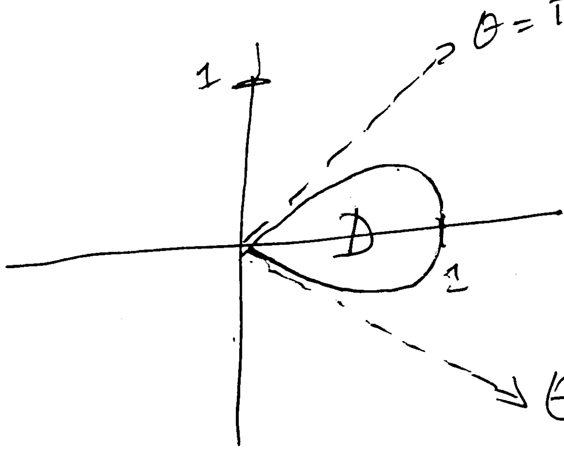
$r = \cos 2\theta, \quad -\pi/4 \le \theta \le \pi/4.$

Let $z = f(x,y) = \sqrt{x^2+y^2}$.

Determine $\iint_D f(x,y)\,dA = \iint_D \sqrt{x^2+y^2}\,dA$.

Sol'n: See next page.

Sol'n: Region D is shown below:



$r = \cos 2\theta, \quad -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

Region D is a Type II polar region:

$D: (r, \theta)$ with $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$ and $0 \le r \le \cos 2\theta$.

Then, $$\iint_D \sqrt{x^2+y^2}\, dA = \int_{-\pi/4}^{\pi/4} \left(\sqrt{r^2}\right) r\, dr\, d\theta$$

$r^2 = x^2+y^2$. The special r-factor.

$$= \int_{-\pi/4}^{\pi/4} r \cdot r\, dr\, d\theta = \int_{-\pi/4}^{\pi/4} r^2\, dr\, d\theta$$

$$= \int_{-\pi/4}^{\pi/4} \left(\left(\tfrac{1}{3} r^3\right) \Big|_{r=0}^{r=\cos 2\theta} \right) d\theta = \int_{-\pi/4}^{\pi/4} \tfrac{1}{3} \cos^3 2\theta\, d\theta$$

$$= \frac{1}{3} \int_{-\pi/4}^{\pi/4} (\cos^2 2\theta)(\cos 2\theta)\, d\theta$$

$\cos^2 2\theta = \underline{1 - \sin^2 2\theta}$

$u = \sin 2\theta$
$du = 2\cos 2\theta\, d\theta$ ~~$2\cos\theta\, d\theta$~~ $= \frac{1}{2} du$
When $\theta = -\frac{\pi}{4}$
$u = \sin\left(-\frac{\pi}{2}\right)$
$u = -1$
When $\theta = \pi/4$,
$u = \sin(\pi/2) = 1$

$$= \frac{1}{2} \cdot \frac{1}{3} \int_{-1}^{1} (1 - u^2)\, du = \frac{1}{6}\left(u - \tfrac{1}{3}u^3\right)\Big|_{-1}^{1}$$

(Continued on next page)

So, $\iint_D \sqrt{x^2+y^2}\,dA = \cdots = \frac{1}{6}\left(u - \frac{1}{3}u^3\right)\Big|_{-1}^{1}$

$= \frac{1}{6}\left(\left(1-\frac{1}{3}\right) - \left(-1-\left(-\frac{1}{3}\right)\right)\right)$

$= \frac{1}{6}\left(\frac{2}{3} - \left(-\frac{2}{3}\right)\right) = \frac{1}{6}\left(\frac{4}{3}\right) = \frac{1}{3}\left(\frac{2}{3}\right) = \frac{2}{9}.$

So, $\iint_D \sqrt{x^2+y^2}\,dA = 2/9$

Part III: $\int_0^1 \int_0^x (4x^2+5)^{\frac{1}{2}}\,dy\,dx$

$= \int_0^1 \left(\int_0^x (4x^2+5)^{\frac{1}{2}}\,dy \right) dx$

$= \int_0^1 \left((4x^2+5)^{\frac{1}{2}}(y) \Big|_{y=0}^{y=x} \right) dx$

$= \int_0^1 [(4x^2+5)^{\frac{1}{2}} \cdot x - 0]\,dx = \int_0^1 (4x^2+5)^{\frac{1}{2}} x\,dx$

Let $u = 4x^2+5$
$du = 8x\,dx$
$x\,dx = \frac{1}{8}du$
When $x=0$, $u=5$
When $x=1$, $u=9$

$= \frac{1}{8}\int_5^9 u^{\frac{1}{2}}\,du = \frac{1}{8}\left(\frac{2}{3}u^{3/2}\right)\Big|_5^9$

$= \left(\frac{1}{12} u\sqrt{u}\right)\Big|_5^9$

$= \frac{1}{12}(9\times 3) - \frac{1}{12}5\sqrt{5} = \frac{1}{12}(27 - 5\sqrt{5})$ square units