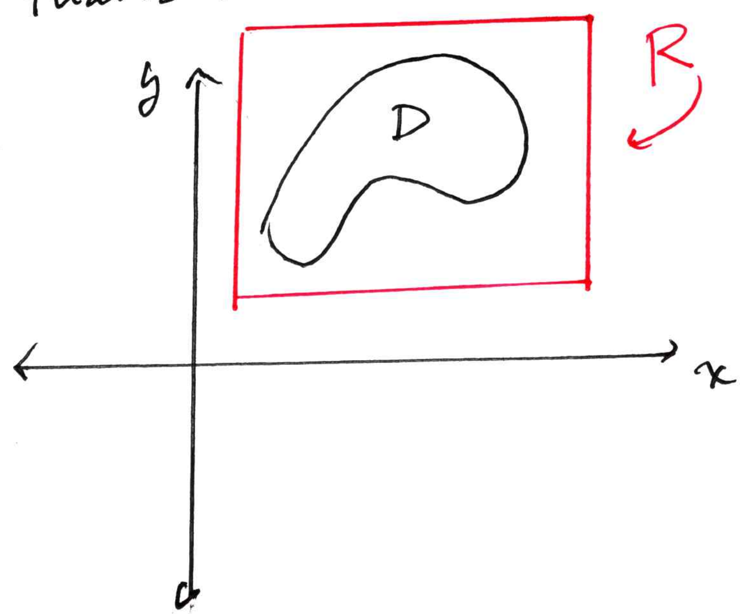
PROJECTED WRITTEN NOTES FROM THE M 408D (2-3) LECTURE ON WEDNESDAY, APRIL 27, 2022, ON SECTION 15.2 – THE DOUBLE INTEGRAL OVER MORE GENERAL REGIONS

CLASS #40

Sec. 15.2 Defining The Double Integral over more general regions D.

Let $z = f(x,y)$ be given. Suppose that D is a bounded region in the xy plane that is the contained in the domain of f.



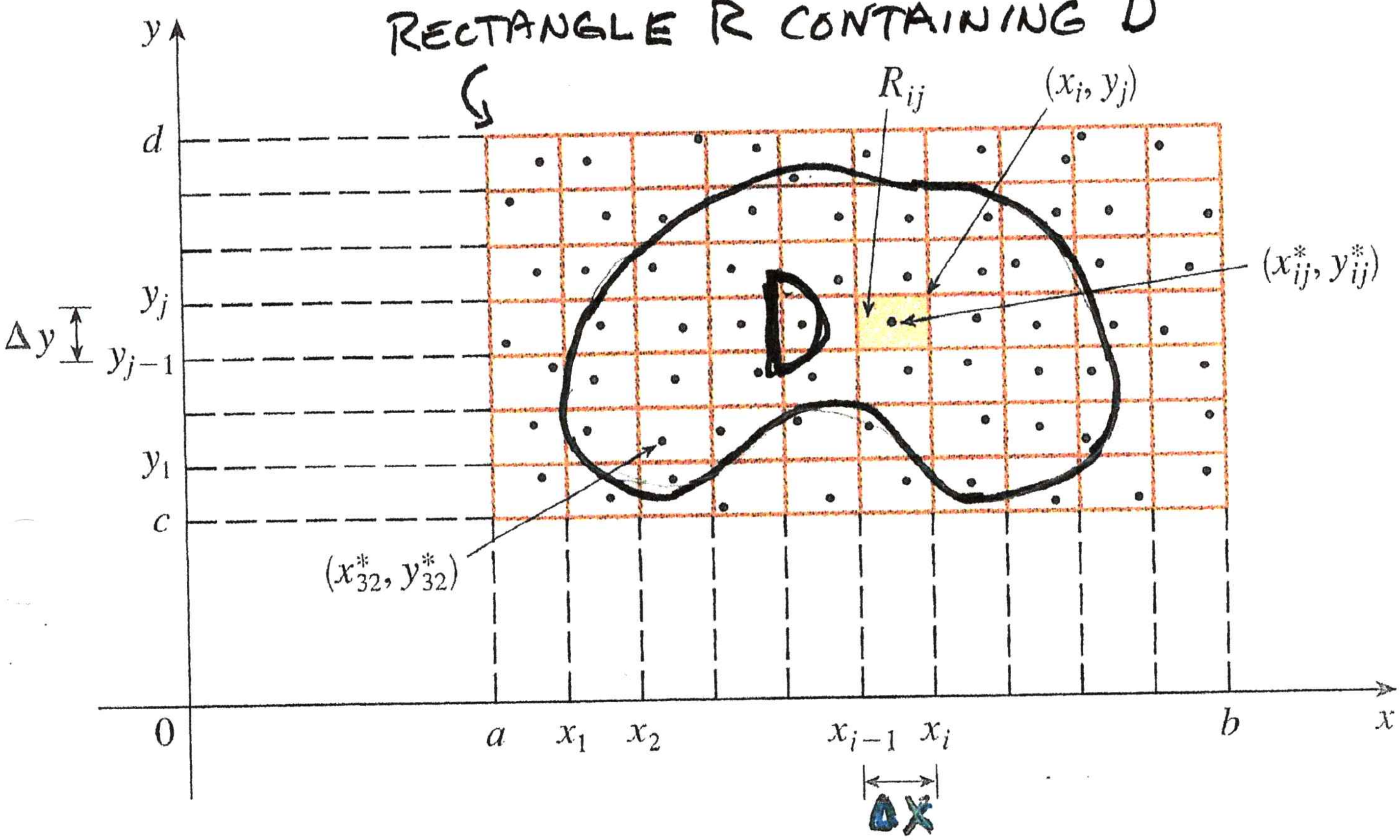
Dr. Slainly's Definition of the Double Integral of $z = f(x,y)$ over Region D:

$$\iint_D f(x,y)\,dA = \lim_{\substack{m\to\infty \\ n\to\infty}} \left(\underbrace{\sum_{i's} \sum_{j's} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}}_{} \right)$$

Only for those points (x_{ij}^*, y_{ij}^*) such that (x_{ij}^*, y_{ij}^*) is in Region D.

How do we determine what number this is?

$$\iint_D f(x,y)\,dA = \lim_{\substack{m\to\infty \\ n\to\infty}} \left(\underbrace{\sum_{i\text{'s}} \sum_{j\text{'s}}}_{\text{where the restrictions below hold.}} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij} \right)$$



HERE, the Double Riemann Sum $R_{m,n}$

is $R_{m,n} = \underbrace{\sum_{i\text{'s}} \sum_{j\text{'s}}}_{\substack{\text{Only for those} \\ \text{selected points } (x_{ij}^*,\, y_{ij}^*) \\ \text{such that } (x_{ij}^*,\, y_{ij}^*) \text{ is} \\ \text{in Region } D.}} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}$

We can use Iterated Integrals

(1) If the region D is a Type I Region

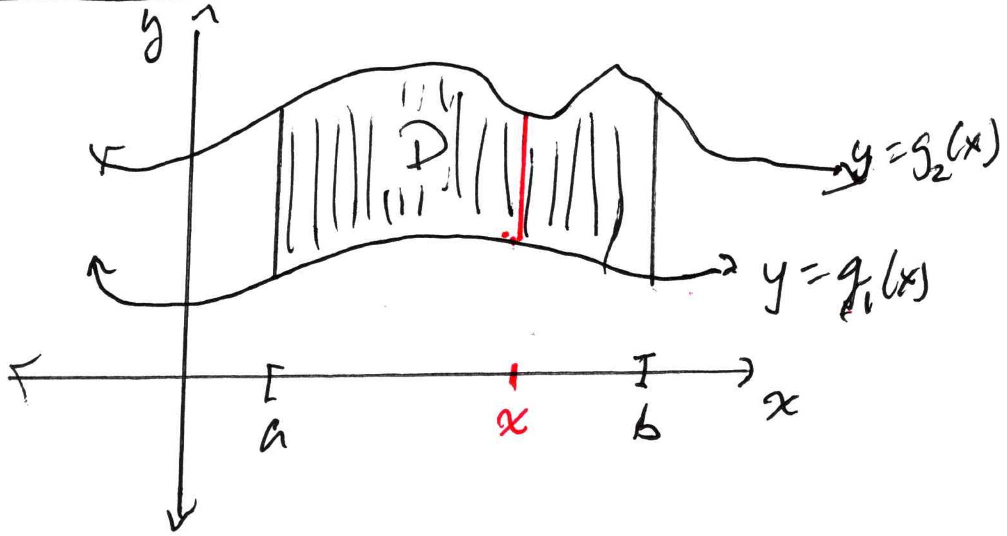
or (2) If the region D is a Type II Region.

Type I Regions

For all x with

$a \le x \le b$,

$g_1(x) \le g_2(x)$



A Type I Description of Region D:

$D = \{ (x, y)$ such that $a \le x \le b$ and $g_1(x) \le y \le g_2(x)$.

SHORTHAND: $D: a \le x \le b$ and $g_1(x) \le y \le g_2(x)$.

When D is a Type I Region

$$\iint_D f(x,y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$$

$y = g_2(x)$

$y = g_1(x)$

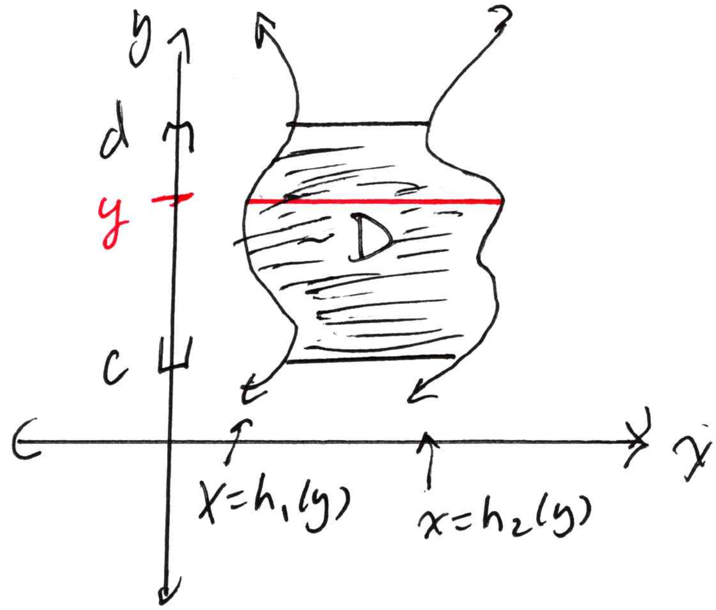
"$dy\, dx$" is for Type I Regions.

Type II Regions

For all y with

$c \le y \le d$,

$h_1(y) \le h_2(y)$



A Type II Description of Region D:

$D = \{(x,y) \text{ such that } c \le y \le d \text{ and } h_1(y) \le x \le h_2(y)\}$

SHORTHAND: $D: c \le y \le d$ and $h_1(y) \le x \le h_2(y)$

WHEN D is a Type II Region,

$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

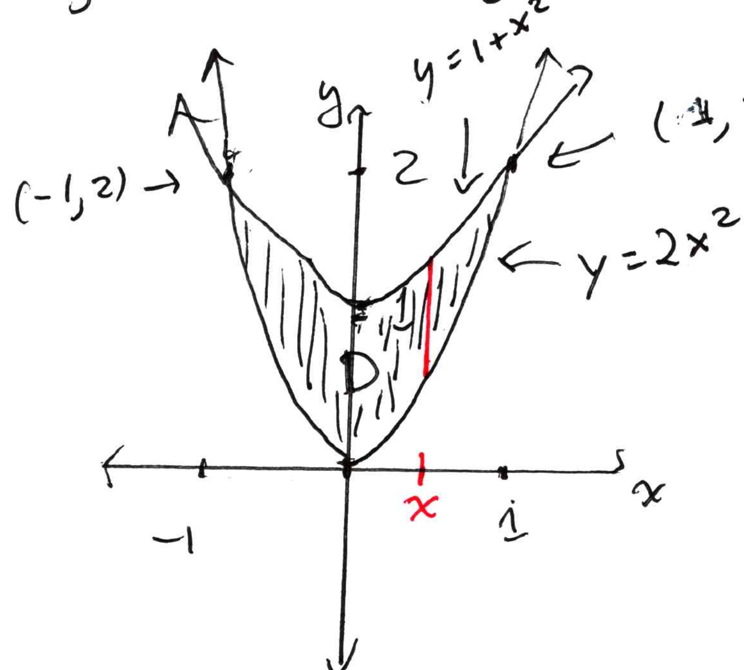
$x = h_2(y)$

$x = h_1(y)$

"$dx\,dy$" is for Type II Regions.

Problem: Determine $\iint_D (x+2y)\,dA$ where

D is the region bounded by $y = 2x^2$ and $y = 1+x^2$.



A Type I Description of D:

$$-1 \le x \le 1 \text{ and } 2x^2 \le y \le 1+x^2.$$

$y = 2x^2$ $\qquad$ $y = 1+x^2$

$$\iint_D (x+2y)\,dA = \int_{-1}^{1}\int_{2x^2}^{1+x^2} (x+2y)\,dy\,dx$$

$$= \int_{-1}^{1}\left((xy + y^2)\Big|_{2x^2}^{1+x^2} \right) dx$$

$$= \int_{-1}^{1}\left(\left(x(1+x^2) + (1+x^2)^2\right) - \left(x(2x^2) + (2x^2)^2\right) \right) dx$$

$$= \int_{-1}^{1} (-3x^4 - x^3 + 2x^2 + x + 1)\,dx$$

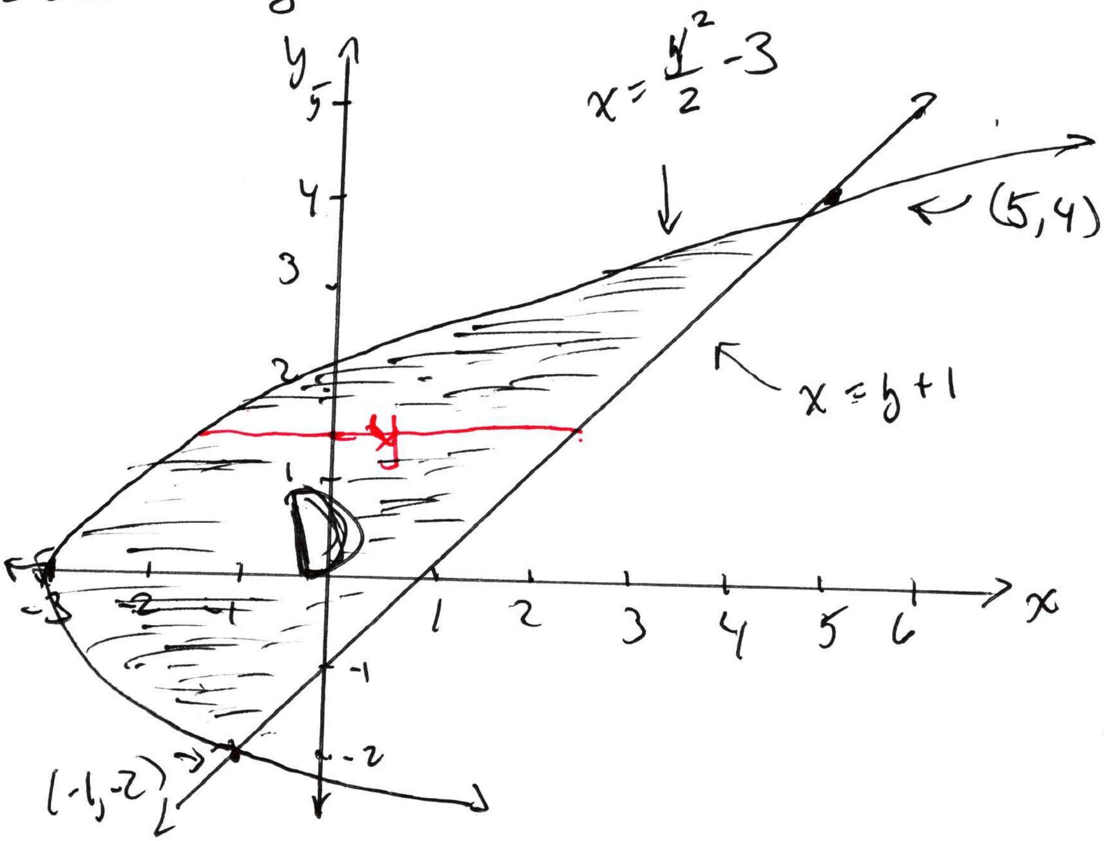
$$= \left(-\frac{3}{5}x^5 - \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 + x\right)\Big|_{-1}^{1}$$

$$= \ldots = \frac{32}{15} = 2\,{}^{2}/_{15}$$

$$\iint_D (x+2y)\,dA = \frac{32}{15} = 2\,{}^{2}/_{15}$$

A Type II EXAMPLE:

FIND $\iint_D (xy)\,dA$ where D is the region bounded by $x = \frac{y^2}{2} - 3$ and $x = y + 1$



A Type II DESCRIPTION OF D:

$$-2 \le y \le 4 \text{ and } \frac{y^2}{2} - 3 \le x \le y + 1$$

$$\iint_D (xy)\,dA = \int_{-2}^{4} \int_{\frac{y^2}{2}-3}^{y+1} (xy)\,dx\,dy = \dots = 36$$

"dx dy" is for Type II Regions.