SOLUTION TO PROBLEM #24 of Section 11.4

DETERMINE WHETHER THE SERIES $\sum_{n=1}^{\infty} \frac{n+3^n}{n+2^n}$

CONVERGES OR DIVERGES.

SOL'N: WE USE THE LIMIT COMPARISON TEST WITH

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{3^n}{2^n} = \sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n \text{ which is a DIVERGENT}$$

GEOMETRIC SERIES SINCE $\frac{3}{2} > 1$.

$$\lim_{n\to\infty} \frac{\frac{n+3^n}{n+2^n}}{\frac{3^n}{2^n}} = \lim_{n\to\infty} \frac{n+3^n}{n+2^n} \cdot \frac{2^n}{3^n} = \lim_{n\to\infty} \frac{n2^n+6^n}{n3^n+6^n}$$

$$\lim_{n\to\infty} \frac{\cancel{6^n}\left(\frac{n}{3^n}+1\right)}{\cancel{6^n}\left(n/2^n+1\right)} = \frac{0+1}{0+1} = 1 \text{ Since } \lim_{n\to\infty} \frac{n}{3^n} = \lim_{n\to\infty} \frac{n}{2^n} = 0$$

as shown below. Since $1 \neq 0$ and $1 \neq \infty$,

$\sum_{n=1}^{\infty} \frac{n+3^n}{n+2^n}$ is also DIVERGENT by the LIMIT COMPARISON TEST.

VERIFICATION THAT $\lim_{n\to\infty} n/3^n = 0$:

$$\lim_{n\to\infty} n/3^n = \underbrace{\lim_{n\to\infty} \frac{x}{3^x}}_{\text{FORM} = \frac{\infty}{\infty}} \overset{L.R.}{=} \lim_{n\to\infty} \frac{\overset{\to 1}{1}}{\underset{\to\infty}{3^x \ln 3}} = 0$$

similarly, $\lim_{n\to\infty} n/2^n = 0$.