Approximating the Sum $s = \sum_{n=1}^{\infty} a_n$

When the Comparison Test Applies

Suppose $s = \sum_{n=1}^{\infty} a_n$ and $t = \sum_{n=1}^{\infty} b_n$ are convergent series with $0 < a_n \leq b_n$ for all $n \geq N$.

Result: We have two sequences of partial sums:

$s_1, s_2, s_3, s_4, \ldots$, with $\lim_{n \to \infty} s_n = s = \sum_{n=1}^{\infty} a_n$

and

$t_1, t_2, t_3, t_4, \ldots$, with $\lim_{n \to \infty} t_n = t = \sum_{n=1}^{\infty} b_n$.

So, for each n, s_n and t_n can be used as approximations of sums s and t; That is,

$s_n \approx s$ and $t_n \approx t$, with errors

$R_n = s - s_n$ and $T_n = t - t_n$, respectively.

Then, for each n, ERROR $R_n \leq$ ERROR T_n.

Furthermore, if $t = \sum_{n=1}^{\infty} b_n$ has a function $y = f(x)$ which satifies conditions ①, ②, ③, ④ such that The Integral Test Applies to $\sum_{n=1}^{\infty} b_n$, Then,

Considering $s_n \approx s$,

The ERROR $R_n \leq T_n \leq \int_n^{\infty} f(x)\,dx$.