SUMMING SUMMATIONS AND USING OTHER OPERATIONS

FACT: (For c a constant)

When $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent,

then, so too are:

$$\sum_{n=1}^{\infty} (c a_n), \quad \sum_{n=1}^{\infty} (a_n + b_n), \quad \sum_{n=1}^{\infty} (a_n - b_n)$$

AND $\sum_{n=1}^{\infty} (c a_n) = c\left(\sum_{n=1}^{\infty} a_n\right)$

$$\sum_{n=1}^{\infty} (a_n + b_n) = \left(\sum_{n=1}^{\infty} a_n\right) + \left(\sum_{n=1}^{\infty} b_n\right)$$

$$\sum_{n=1}^{\infty} (a_n - b_n) = \left(\sum_{n=1}^{\infty} a_n\right) - \left(\sum_{n=1}^{\infty} b_n\right)$$

EXAMPLE: $\sum_{n=0}^{\infty} \left(\frac{5^n - 3^n}{15^n}\right)$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{5^n}{15^n}\right) - \left(\frac{3^n}{15^n}\right)\right] = \sum_{n=0}^{\infty} \left[\left(\frac{5}{15}\right)^n - \left(\frac{3}{15}\right)^n\right]$$

$$= \left(\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n\right) - \left(\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n\right)$$

$$= \frac{1}{1 - 1/3} - \frac{1}{1 - 1/5} = \frac{3}{2} - \frac{5}{4} = \frac{6}{4} - \frac{5}{4} = \underline{\underline{\frac{1}{4}}}$$