HW #11, Sec 14.2 Solutions

4. We make a table of values of

$f(x,y) = \dfrac{2xy}{x^2+2y^2}$ for a set of (x,y)

points near the origin.

x \ y	−0.3	−0.2	−0.1	0	0.1	0.2	0.3
−0.3	0.667	0.706	0.545	0.000	−0.545	−0.706	−0.667
−0.2	0.545	0.667	0.667	0.000	−0.667	−0.667	−0.545
−0.1	0.316	0.444	0.667	0.000	−0.667	−0.444	−0.316
0	0.000	0.000	0.000		0.000	0.000	0.000
0.1	−0.316	−0.444	−0.667	0.000	0.667	0.444	0.316
0.2	−0.545	−0.667	−0.667	0.000	0.667	0.667	0.545
0.3	−0.667	−0.706	−0.545	0.000	0.545	0.706	0.667

It appears from the table that the values of $f(x,y)$ are not approaching a single value as (x,y) approaches the origin. For verification, if we first approach $(0,0)$ along the x-axis, we have $f(x,0)=0$, so $f(x,y)\to 0$. But if we approach $(0,0)$ along the line $y=x$, $f(x,x)=\dfrac{2x^2}{x^2+2x^2}=\dfrac{2}{3}$ $(x\neq 0)$, so $f(x,y)\to\frac{2}{3}$. Since f approaches different values along different paths to the origin, this limit does not exist.

5. $f(x,y)=x^2y^3-4y^2$ is a polynomial, and hence continuous, so we can find the limit by direct substitution:

$\lim\limits_{(x,y)\to(3,2)} f(x,y) = f(3,2) = (3)^2(2)^3 - 4(2)^2 = 56.$

6. $f(x,y)=x^2y+3xy^2+4$ is a polynomial, and hence continuous, so we can find the limit by direct substitution:

$\lim\limits_{(x,y)\to(5,-2)} f(x,y) = f(5,-2) = 5^2(-2)+3(5)(-2)^2+4 = 14.$

7. $f(x,y)=\dfrac{x^2y-xy^3}{x-y+2}$ is a rational function, and hence, continuous on its domain. $(-3,1)$ is in the domain of f, so we can find the limit by direct substitution: $\lim\limits_{(x,y)\to(-3,1)} f(x,y) = f(-3,1) = \dfrac{(-3)^2(1)-(-3)(1)^3}{-3-1+2} = \dfrac{12}{-2} = -6.$

8. $f(x,y)=\dfrac{x^2y+xy^2}{x^2-y^2}$ is a rational function, and hence, continuous on its domain. $(2,-1)$ is in the domain of f, so we can find the limit by direct substitution: $\lim\limits_{(x,y)\to(2,-1)} f(x,y) = f(2,-1) = \dfrac{(2)^2(-1)+(2)(-1)^2}{(2)^2-(-1)^2} = -\dfrac{2}{3}.$

9. $x-y$ is a polynomial and therefore continuous. Since $\sin t$ is a continuous function, the composition $\sin(x-y)$ is also continuous. The function y is a polynomial, and hence continuous, and the product of continuous functions is continuous, so $f(x,y)=y\sin(x-y)$ is a continuous function. Then $\lim\limits_{(x,y)\to(\pi,\pi/2)} f(x,y) = f\left(\pi,\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\left(\pi-\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi}{2}.$

10. $2x-y$ is a polynomial and therefore continuous. Since $\sqrt{t}$ is continuous for $t\geq 0$, the composition $\sqrt{2x-y}$ is continuous where $2x-y\geq 0$. The function e^u is continuous everywhere, so the composition $f(x,y)=e^{\sqrt{2x-y}}$ is a continuous function for $2x-y\geq 0$. If $x=3$ and $y=2$ then $2x-y\geq 0$, so $\lim\limits_{(x,y)\to(3,2)} f(x,y) = f(3,2) = e^{\sqrt{2(3)-2}} = e^2.$