HW # 11, Sec 14.3 Solutions

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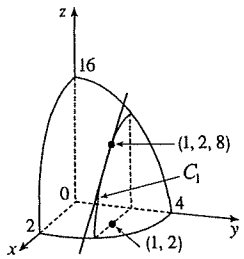
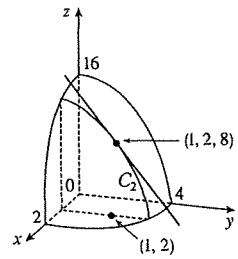
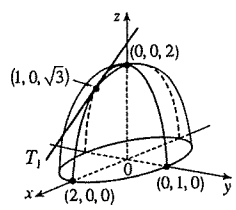
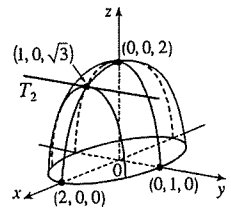
(the curve C_1 in the first figure). The slope of the tangent line to this parabola at $(1, 2, 8)$ is $f_x(1, 2) = -8$. Similarly the plane $x = 1$ intersects the paraboloid in the parabola $z = 12 - y^2$, $x = 1$ (the curve C_2 in the second figure) and the slope of the tangent line at $(1, 2, 8)$ is $f_y(1, 2) = -4$.





8. $f(x, y) = (4 - x^2 - 4y^2)^{1/2} \Rightarrow f_x(x, y) = -x(4 - x^2 - 4y^2)^{-1/2}$ and $f_y(x, y) = -4y(4 - x^2 - 4y^2)^{-1/2} \Rightarrow$ $f_x(1, 0) = -\frac{1}{\sqrt{3}}$, $f_y(1, 0) = 0$. The graph of f is the upper half of the ellipsoid $z^2 + x^2 + 4y^2 = 4$ and the plane $y = 0$ intersects the graph in the semicircle $x^2 + z^2 = 4$, $z \geq 0$ and the slope of the tangent line T_1 to this semicircle at $(1, 0, \sqrt{3})$ is $f_x(1, 0) = -\frac{1}{\sqrt{3}}$. Similarly the plane $x = 1$ intersects the graph in the semi-ellipse $z^2 + 4y^2 = 3$, $z \geq 0$ and the slope of the tangent line T_2 to this semi-ellipse at $(1, 0, \sqrt{3})$ is $f_y(1, 0) = 0$.





9. $f(x, y) = x^4 + 5xy^3 \Rightarrow f_x(x, y) = 4x^3 + 5y^3$, $f_y(x, y) = 0 + 5x \cdot 3y^2 = 15xy^2$

10. $f(x, y) = x^2y - 3y^4 \Rightarrow f_x(x, y) = 2x \cdot y - 0 = 2xy$, $f_y(x, y) = x^2 \cdot 1 - 3 \cdot 4y^3 = x^2 - 12y^3$

11. $g(x, y) = x^3 \sin y \Rightarrow g_x(x, y) = 3x^2 \sin y$, $g_y(x, y) = x^3 \cos y$

12. $g(x, t) = e^{xt} \Rightarrow g_x(x, t) = e^{xt} \cdot t = te^{xt}$, $g_t(x, t) = e^{xt} \cdot x = xe^{xt}$

13. $z = \ln(x + t^2) \Rightarrow \dfrac{\partial z}{\partial x} = \dfrac{1}{x + t^2} \cdot 1 = \dfrac{1}{x + t^2}$, $\dfrac{\partial z}{\partial t} = \dfrac{1}{x + t^2} \cdot 2t = \dfrac{2t}{x + t^2}$

14. $w = \dfrac{u}{v^2} \Rightarrow w_u = \dfrac{1}{v^2} \cdot 1 = \dfrac{1}{v^2}$, $w_v = u \cdot (-2v^{-3}) = -\dfrac{2u}{v^3}$

15. $f(x, y) = ye^{xy} \Rightarrow f_x(x, y) = y \cdot e^{xy} \cdot y = y^2e^{xy}$, $f_y(x, y) = y \cdot e^{xy} \cdot x + e^{xy} \cdot 1 = e^{xy} + xye^{xy}$

16. $g(x, y) = (x^2 + xy)^3 \Rightarrow g_x(x, y) = 3(x^2 + xy)^2(2x + y)$, $g_y(x, y) = 3(x^2 + xy)^2(0 + x) = 3x(x^2 + xy)^2$

17. $g(x, y) = y(x + x^2y)^5 \Rightarrow g_x(x, y) = 5y(x + x^2y)^4(1 + 2xy)$,

$g_y(x, y) = y \cdot 5(x + x^2y)^4 \cdot x^2 + (x + x^2y)^5 \cdot 1 = 5x^2y(x + x^2y)^4 + (x + x^2y)^5$

18. $f(x, y) = \dfrac{x}{(x + y)^2} \Rightarrow f_x(x, y) = \dfrac{(x + y)^2(1) - (x)(2)(x + y)}{[(x + y)^2]^2} = \dfrac{x + y - 2x}{(x + y)^3} = \dfrac{y - x}{(x + y)^3}$,

$f_y(x, y) = \dfrac{(x + y)^2(0) - (x)(2)(x + y)}{[(x + y)^2]^2} = -\dfrac{2x}{(x + y)^3}$

19. $f(x,y) = \dfrac{ax+by}{cx+dy} \quad \Rightarrow \quad f_x(x,y) = \dfrac{(cx+dy)(a)-(ax+by)(c)}{(cx+dy)^2} = \dfrac{(ad-bc)y}{(cx+dy)^2}$,

$f_y(x,y) = \dfrac{(cx+dy)(b)-(ax+by)(d)}{(cx+dy)^2} = \dfrac{(bc-ad)x}{(cx+dy)^2}$

20. $w = \dfrac{e^v}{u+v^2} \quad \Rightarrow \quad \dfrac{\partial w}{\partial u} = \dfrac{0(u+v^2)-e^v(1)}{(u+v^2)^2} = -\dfrac{e^v}{(u+v^2)^2}, \quad \dfrac{\partial w}{\partial v} = \dfrac{e^v(u+v^2)-e^v(2v)}{(u+v^2)^2} = \dfrac{e^v(u+v^2-2v)}{(u+v^2)^2}$

21. $g(u,v) = (u^2v-v^3)^5 \quad \Rightarrow \quad g_u(u,v) = 5(u^2v-v^3)^4 \cdot 2uv = 10uv(u^2v-v^3)^4$,

$g_v(u,v) = 5(u^2v-v^3)^4(u^2-3v^2) = 5(u^2-3v^2)(u^2v-v^3)^4$

22. $u(r,\theta) = \sin(r\cos\theta) \quad \Rightarrow \quad u_r(r,\theta) = \cos(r\cos\theta)\cdot\cos\theta = \cos\theta\cos(r\cos\theta)$,

$u_\theta(r,\theta) = \cos(r\cos\theta)(-r\sin\theta) = -r\sin\theta\cos(r\cos\theta)$

23. $R(p,q) = \tan^{-1}(pq^2) \quad \Rightarrow \quad R_p(p,q) = \dfrac{1}{1+(pq^2)^2}\cdot q^2 = \dfrac{q^2}{1+p^2q^4}, \quad R_q(p,q) = \dfrac{1}{1+(pq^2)^2}\cdot 2pq = \dfrac{2pq}{1+p^2q^4}$

24. $f(x,y) = x^y \quad \Rightarrow \quad f_x(x,y) = yx^{y-1}, \; f_y(x,y) = x^y \ln x$

25. $F(x,y) = \displaystyle\int_y^x \cos(e^t)\,dt \quad \Rightarrow \quad F_x(x,y) = \frac{\partial}{\partial x}\int_y^x \cos(e^t)\,dt = \cos(e^x)$ by the Fundamental Theorem of Calculus, Part 1;

$F_y(x,y) = \dfrac{\partial}{\partial y}\displaystyle\int_y^x \cos(e^t)\,dt = \frac{\partial}{\partial y}\left[-\int_x^y \cos(e^t)\,dt\right] = -\frac{\partial}{\partial y}\int_x^y \cos(e^t)\,dt = -\cos(e^y)$.

26. $F(\alpha,\beta) = \displaystyle\int_\alpha^\beta \sqrt{t^3+1}\,dt \quad \Rightarrow$

$F_\alpha(\alpha,\beta) = \dfrac{\partial}{\partial\alpha}\displaystyle\int_\alpha^\beta \sqrt{t^3+1}\,dt = \frac{\partial}{\partial\alpha}\left[-\int_\beta^\alpha \sqrt{t^3+1}\,dt\right] = -\frac{\partial}{\partial\alpha}\int_\beta^\alpha \sqrt{t^3+1}\,dt = -\sqrt{\alpha^3+1}$ by the Fundamental Theorem of Calculus, Part 1; $F_\beta(\alpha,\beta) = \dfrac{\partial}{\partial\beta}\displaystyle\int_\alpha^\beta \sqrt{t^3+1}\,dt = \sqrt{\beta^3+1}$.

27. $f(x,y,z) = x^3yz^2+2yz \quad \Rightarrow \quad f_x(x,y,z) = 3x^2yz^2, \; f_y(x,y,z) = x^3z^2+2z, \; f_z(x,y,z) = 2x^3yz+2y$

28. $f(x,y,z) = xy^2e^{-xz} \quad \Rightarrow \quad f_x(x,y,z) = y^2\left[x\cdot e^{-xz}(-z)+e^{-xz}\cdot 1\right] = (1-xz)y^2e^{-xz}, \; f_y(x,y,z) = 2xye^{-xz}$,

$f_z(x,y,z) = xy^2e^{-xz}(-x) = -x^2y^2e^{-xz}$

29. $w = \ln(x+2y+3z) \quad \Rightarrow \quad \dfrac{\partial w}{\partial x} = \dfrac{1}{x+2y+3z}, \; \dfrac{\partial w}{\partial y} = \dfrac{2}{x+2y+3z}, \; \dfrac{\partial w}{\partial z} = \dfrac{3}{x+2y+3z}$

30. $w = y\tan(x+2z) \quad \Rightarrow \quad \dfrac{\partial w}{\partial x} = y\,[\sec^2(x+2z)](1) = y\sec^2(x+2z), \; \dfrac{\partial w}{\partial y} = \tan(x+2z)$,

$\dfrac{\partial w}{\partial z} = y\,[\sec^2(x+2z)](2) = 2y\sec^2(x+2z)$

31. $p = \sqrt{t^4+u^2\cos v} \quad \Rightarrow \quad \dfrac{\partial p}{\partial t} = \frac{1}{2}(t^4+u^2\cos v)^{-1/2}(4t^3) = \dfrac{2t^3}{\sqrt{t^4+u^2\cos v}}$,

$\dfrac{\partial p}{\partial u} = \frac{1}{2}(t^4+u^2\cos v)^{-1/2}(2u\cos v) = \dfrac{u\cos v}{\sqrt{t^4+u^2\cos v}}, \; \dfrac{\partial p}{\partial v} = \frac{1}{2}(t^4+u^2\cos v)^{-1/2}[u^2(-\sin v)] = -\dfrac{u^2\sin v}{2\sqrt{t^4+u^2\cos v}}$

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32. $u = x^{y/z} \quad \Rightarrow \quad u_x = \dfrac{y}{z}\, x^{(y/z)-1},\ u_y = x^{y/z}\ln x \cdot \dfrac{1}{z} = \dfrac{x^{y/z}}{z}\ln x,\ \ u_z = x^{y/z}\ln x \cdot \dfrac{-y}{z^2} = -\dfrac{yx^{y/z}}{z^2}\ln x$

33. $h(x,y,z,t) = x^2 y\cos(z/t) \quad \Rightarrow \quad h_x(x,y,z,t) = 2xy\cos(z/t),\ h_y(x,y,z,t) = x^2\cos(z/t),$

$h_z(x,y,z,t) = -x^2 y\sin(z/t)(1/t) = (-x^2y/t)\sin(z/t),\ h_t(x,y,z,t) = -x^2y\sin(z/t)(-zt^{-2}) = (x^2yz/t^2)\sin(z/t)$

34. $\phi(x,y,z,t) = \dfrac{\alpha x + \beta y^2}{\gamma z + \delta t^2} \quad \Rightarrow \quad \phi_x(x,y,z,t) = \dfrac{1}{\gamma z + \delta t^2}(\alpha) = \dfrac{\alpha}{\gamma z + \delta t^2},$

$\phi_y(x,y,z,t) = \dfrac{1}{\gamma z + \delta t^2}(2\beta y) = \dfrac{2\beta y}{\gamma z + \delta t^2},\ \phi_z(x,y,z,t) = \dfrac{(\gamma z + \delta t^2)(0) - (\alpha x + \beta y^2)(\gamma)}{(\gamma z + \delta t^2)^2} = \dfrac{-\gamma(\alpha x + \beta y^2)}{(\gamma z + \delta t^2)^2},$

$\phi_t(x,y,z,t) = \dfrac{(\gamma z + \delta t^2)(0) - (\alpha x + \beta y^2)(2\delta t)}{(\gamma z + \delta t^2)^2} = -\dfrac{2\delta t(\alpha x + \beta y^2)}{(\gamma z + \delta t^2)^2}$

35. $u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$. For each $i = 1, \ldots, n$, $u_{x_i} = \frac{1}{2}\left(x_1^2 + x_2^2 + \cdots + x_n^2\right)^{-1/2}(2x_i) = \dfrac{x_i}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}}$.

36. $u = \sin(x_1 + 2x_2 + \cdots + nx_n)$. For each $i = 1, \ldots, n$, $u_{x_i} = i\cos(x_1 + 2x_2 + \cdots + nx_n)$.

37. $R(s,t) = te^{s/t} \quad \Rightarrow \quad R_t(s,t) = t \cdot e^{s/t}(-s/t^2) + e^{s/t} \cdot 1 = \left(1 - \dfrac{s}{t}\right)e^{s/t}$, so $R_t(0,1) = \left(1 - \dfrac{0}{1}\right)e^{0/1} = 1$.

38. $f(x,y) = y\sin^{-1}(xy) \quad \Rightarrow \quad f_y(x,y) = y \cdot \dfrac{1}{\sqrt{1-(xy)^2}}(x) + \sin^{-1}(xy) \cdot 1 = \dfrac{xy}{\sqrt{1-x^2y^2}} + \sin^{-1}(xy),$

so $f_y\left(1, \frac{1}{2}\right) = \dfrac{1 \cdot \frac{1}{2}}{\sqrt{1 - 1^2\left(\frac{1}{2}\right)^2}} + \sin^{-1}\left(1 \cdot \frac{1}{2}\right) = \dfrac{\frac{1}{2}}{\sqrt{\frac{3}{4}}} + \sin^{-1}\frac{1}{2} = \frac{1}{\sqrt{3}} + \frac{\pi}{6}$.

39. $f(x,y,z) = \ln\dfrac{1 - \sqrt{x^2+y^2+z^2}}{1 + \sqrt{x^2+y^2+z^2}} \quad \Rightarrow$

$$f_y(x,y,z) = \frac{1}{\dfrac{1 - \sqrt{x^2+y^2+z^2}}{1 + \sqrt{x^2+y^2+z^2}}} \cdot \frac{\left(1 + \sqrt{x^2+y^2+z^2}\right)\left(-\frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot 2y\right) - \left(1 - \sqrt{x^2+y^2+z^2}\right)\left(\frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot 2y\right)}{\left(1 + \sqrt{x^2+y^2+z^2}\right)^2}$$

$$= \frac{1 + \sqrt{x^2+y^2+z^2}}{1 - \sqrt{x^2+y^2+z^2}} \cdot \frac{-y(x^2+y^2+z^2)^{-1/2}\left(1 + \sqrt{x^2+y^2+z^2} + 1 - \sqrt{x^2+y^2+z^2}\right)}{\left(1 + \sqrt{x^2+y^2+z^2}\right)^2}$$

$$= \frac{-y(x^2+y^2+z^2)^{-1/2}(2)}{\left(1 - \sqrt{x^2+y^2+z^2}\right)\left(1 + \sqrt{x^2+y^2+z^2}\right)} = \frac{-2y}{\sqrt{x^2+y^2+z^2}\,[1 - (x^2+y^2+z^2)]}$$

so $f_y(1,2,2) = \dfrac{-2(2)}{\sqrt{1^2+2^2+2^2}\,[1 - (1^2+2^2+2^2)]} = \dfrac{-4}{\sqrt{9}\,(1-9)} = \dfrac{1}{6}$.

40. $f(x,y,z) = x^{yz} \Rightarrow f_z(x,y,z) = (x^{yz}\ln x)(y) = yx^{yz}\ln x$, so $f_z(e,1,0) = 1e^{(1)(0)}\ln e = 1$.

41. $x^2+2y^2+3z^2=1 \Rightarrow \dfrac{\partial}{\partial x}(x^2+2y^2+3z^2) = \dfrac{\partial}{\partial x}(1) \Rightarrow 2x+0+6z\dfrac{\partial z}{\partial x} = 0 \Rightarrow 6z\dfrac{\partial z}{\partial x} = -2x \Rightarrow$

$\dfrac{\partial z}{\partial x} = \dfrac{-2x}{6z} = -\dfrac{x}{3z}$, and $\dfrac{\partial}{\partial y}(x^2+2y^2+3z^2) = \dfrac{\partial}{\partial y}(1) \Rightarrow 0+4y+6z\dfrac{\partial z}{\partial y} = 0 \Rightarrow 6z\dfrac{\partial z}{\partial y} = -4y \Rightarrow$

$\dfrac{\partial z}{\partial y} = \dfrac{-4y}{6z} = -\dfrac{2y}{3z}$.

42. $x^2-y^2+z^2-2z=4 \Rightarrow \dfrac{\partial}{\partial x}(x^2-y^2+z^2-2z) = \dfrac{\partial}{\partial x}(4) \Rightarrow 2x-0+2z\dfrac{\partial z}{\partial x} - 2\dfrac{\partial z}{\partial x} = 0 \Rightarrow$

$(2z-2)\dfrac{\partial z}{\partial x} = -2x \Rightarrow \dfrac{\partial z}{\partial x} = \dfrac{-2x}{2z-2} = \dfrac{x}{1-z}$, and $\dfrac{\partial}{\partial y}(x^2-y^2+z^2-2z) = \dfrac{\partial}{\partial y}(4) \Rightarrow$

$0-2y+2z\dfrac{\partial z}{\partial y} - 2\dfrac{\partial z}{\partial y} = 0 \Rightarrow (2z-2)\dfrac{\partial z}{\partial y} = 2y \Rightarrow \dfrac{\partial z}{\partial y} = \dfrac{2y}{2z-2} = \dfrac{y}{z-1}$.

43. $e^z = xyz \Rightarrow \dfrac{\partial}{\partial x}(e^z) = \dfrac{\partial}{\partial x}(xyz) \Rightarrow e^z\dfrac{\partial z}{\partial x} = y\left(x\dfrac{\partial z}{\partial x} + z\cdot 1\right) \Rightarrow e^z\dfrac{\partial z}{\partial x} - xy\dfrac{\partial z}{\partial x} = yz \Rightarrow$

$(e^z - xy)\dfrac{\partial z}{\partial x} = yz$, so $\dfrac{\partial z}{\partial x} = \dfrac{yz}{e^z - xy}$.

$\dfrac{\partial}{\partial y}(e^z) = \dfrac{\partial}{\partial y}(xyz) \Rightarrow e^z\dfrac{\partial z}{\partial y} = x\left(y\dfrac{\partial z}{\partial x} + z\cdot 1\right) \Rightarrow e^z\dfrac{\partial z}{\partial y} - xy\dfrac{\partial z}{\partial y} = xz \Rightarrow (e^z - xy)\dfrac{\partial z}{\partial y} = xz$, so

$\dfrac{\partial z}{\partial y} = \dfrac{xz}{e^z - xy}$.

44. $yz + x\ln y = z^2 \Rightarrow \dfrac{\partial}{\partial x}(yz + x\ln y) = \dfrac{\partial}{\partial x}(z^2) \Rightarrow y\dfrac{\partial z}{\partial x} + \ln y = 2z\dfrac{\partial z}{\partial x} \Rightarrow \ln y = 2z\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial x} \Rightarrow$

$\ln y = (2z-y)\dfrac{\partial z}{\partial x}$, so $\dfrac{\partial z}{\partial x} = \dfrac{\ln y}{2z-y}$.

$\dfrac{\partial}{\partial y}(yz + x\ln y) = \dfrac{\partial}{\partial y}(z^2) \Rightarrow y\dfrac{\partial z}{\partial y} + z\cdot 1 + x\cdot\dfrac{1}{y} = 2z\dfrac{\partial z}{\partial y} \Rightarrow z + \dfrac{x}{y} = 2z\dfrac{\partial z}{\partial y} - y\dfrac{\partial z}{\partial y} \Rightarrow$

$z + \dfrac{x}{y} = (2z-y)\dfrac{\partial z}{\partial y}$, so $\dfrac{\partial z}{\partial y} = \dfrac{z + (x/y)}{2z-y} = \dfrac{x+yz}{y(2z-y)}$.

45. (a) $z = f(x) + g(y) \Rightarrow \dfrac{\partial z}{\partial x} = f'(x)$, $\dfrac{\partial z}{\partial y} = g'(y)$

(b) $z = f(x+y)$. Let $u = x+y$. Then $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = \dfrac{df}{du}(1) = f'(u) = f'(x+y)$,

$\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = \dfrac{df}{du}(1) = f'(u) = f'(x+y)$.

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46. (a) $z = f(x)g(y) \quad \Rightarrow \quad \dfrac{\partial z}{\partial x} = f'(x)g(y), \quad \dfrac{\partial z}{\partial y} = f(x)g'(y)$

(b) $z = f(xy)$. Let $u = xy$. Then $\dfrac{\partial u}{\partial x} = y$ and $\dfrac{\partial u}{\partial y} = x$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = \dfrac{df}{du} \cdot y = yf'(u) = yf'(xy)$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = \dfrac{df}{du} \cdot x = xf'(u) = xf'(xy)$.

(c) $z = f\left(\dfrac{x}{y}\right)$. Let $u = \dfrac{x}{y}$. Then $\dfrac{\partial u}{\partial x} = \dfrac{1}{y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{x}{y^2}$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = f'(u)\dfrac{1}{y} = \dfrac{f'(x/y)}{y}$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = f'(u)\left(-\dfrac{x}{y^2}\right) = -\dfrac{xf'(x/y)}{y^2}$.

47. $f(x,y) = x^4y - 2x^3y^2 \quad \Rightarrow \quad f_x(x,y) = 4x^3y - 6x^2y^2, \;\; f_y(x,y) = x^4 - 4x^3y$. Then $f_{xx}(x,y) = 12x^2y - 12xy^2$, $f_{xy}(x,y) = 4x^3 - 12x^2y$, $f_{yx}(x,y) = 4x^3 - 12x^2y$, and $f_{yy}(x,y) = -4x^3$.

48. $f(x,y) = \ln(ax+by) \quad \Rightarrow \quad f_x(x,y) = \dfrac{a}{ax+by} = a(ax+by)^{-1}, \;\; f_y(x,y) = \dfrac{b}{ax+by} = b(ax+by)^{-1}$. Then

$f_{xx}(x,y) = -a(ax+by)^{-2}(a) = -\dfrac{a^2}{(ax+by)^2}, \;\; f_{xy}(x,y) = -a(ax+by)^{-2}(b) = -\dfrac{ab}{(ax+by)^2}$,

$f_{yx}(x,y) = -b(ax+by)^{-2}(a) = -\dfrac{ab}{(ax+by)^2}$, and $f_{yy}(x,y) = -b(ax+by)^{-2}(b) = -\dfrac{b^2}{(ax+by)^2}$.

49. $z = \dfrac{y}{2x+3y} = y(2x+3y)^{-1} \quad \Rightarrow \quad z_x = y(-1)(2x+3y)^{-2}(2) = -\dfrac{2y}{(2x+3y)^2}$,

$z_y = \dfrac{(2x+3y)\cdot 1 - y\cdot 3}{(2x+3y)^2} = \dfrac{2x}{(2x+3y)^2}$. Then $z_{xx} = -2y(-2)(2x+3y)^{-3}(2) = \dfrac{8y}{(2x+3y)^3}$,

$z_{xy} = -\dfrac{(2x+3y)^2\cdot 2 - 2y\cdot 2(2x+3y)(3)}{[(2x+3y)^2]^2} = -\dfrac{(2x+3y)\,(4x+6y-12y)}{(2x+3y)^4} = \dfrac{6y-4x}{(2x+3y)^3}$,

$z_{yx} = \dfrac{(2x+3y)^2\cdot 2 - 2x\cdot 2(2x+3y)(2)}{[(2x+3y)^2]^2} = \dfrac{6y-4x}{(2x+3y)^3}, \;\; z_{yy} = 2x(-2)(2x+3y)^{-3}(3) = -\dfrac{12x}{(2x+3y)^3}$.

50. $T = e^{-2r}\cos\theta \quad \Rightarrow \quad T_r = -2e^{-2r}\cos\theta, \;\; T_\theta = -e^{-2r}\sin\theta$. Then $T_{rr} = -2e^{-2r}(-2)\cos\theta = 4e^{-2r}\cos\theta$,

$T_{r\theta} = 2e^{-2r}\sin\theta, \;\; T_{\theta r} = -e^{-2r}(-2)\sin\theta = 2e^{-2r}\sin\theta, \;\; T_{\theta\theta} = -e^{-2r}\cos\theta$.

51. $v = \sin(s^2-t^2) \quad \Rightarrow \quad v_s = \cos(s^2-t^2)\cdot 2s = 2s\cos(s^2-t^2), \;\; v_t = \cos(s^2-t^2)\cdot(-2t) = -2t\cos(s^2-t^2)$. Then

$v_{ss} = 2s\big[-\sin(s^2-t^2)\cdot 2s\big] + \cos(s^2-t^2)\cdot 2 = 2\cos(s^2-t^2) - 4s^2\sin(s^2-t^2)$,

$v_{st} = 2s\big[-\sin(s^2-t^2)\cdot(-2t)\big] + \cos(s^2-t^2)\cdot 0 = 4st\sin(s^2-t^2)$,

$v_{ts} = -2t\big[-\sin(s^2-t^2)\cdot 2s\big] + \cos(s^2-t^2)\cdot 0 = 4st\sin(s^2-t^2)$,

$v_{tt} = -2t\big[-\sin(s^2-t^2)\cdot(-2t)\big] + \cos(s^2-t^2)\cdot(-2) = -2\cos(s^2-t^2) - 4t^2\sin(s^2-t^2)$.