M408D FALL 2023

HW#11, Sec. 12.5 SOLUTIONS

#31 THREE POINTS on the plane are:

$(0,1,1)$, $(1,0,1)$, and $(1,1,0)$.

We will use $(x_0, y_0, z_0) = (0,1,1)$.

An equation of this plane is

$$z - z_0 = A(x - x_0) + B(y - y_0)$$

Here: $$z - 1 = A(x-0) + B(y-1) \quad (*)$$

We substitute $(x_1, y_1, z_1) = (1,0,1)$ and $(x_2, y_2, z_2) = (1,1,0)$, in the unknowns A and B below:

and we get equations ① and ②

① $1-1 = A(1-0) + B(0-1)$

↳ ① $\underline{\underline{0 = A - B}}$

② $0-1 = A(1-0) + B(1-1)$ $\Rightarrow$ $A = -1$ $\Rightarrow$ From ① $0 = -1 - B \Rightarrow B = -1$

$-1 = A + 0$

The equation above becomes $z - 1 = (-1)(x) + (-1)(y-1)$,

or, $z - 1 = -x - y + 1 \Rightarrow x + y + z = 2$

A Linear Equation of this plane is

$$\boxed{x + y + z = 2.}$$

SEC. 12.5, #33:

#33. THREE POINTS ON THE PLANE ARE:

$(2,1,2)$, $(3,-8,6)$, $(-2,-3,1)$.

We will use $(x_0, y_0, z_0) = (2,1,2)$.

AN EQUATION OF THIS PLANE IS

$$(z-2) = A(x-2) + B(y-1).$$

We substitute $(x_1,y_1,z_1) = (3,-8,6)$ and $(x_2,y_2,z_2) = (-2,-3,1)$ and we get equations ① and ② with unknowns A and B.

① $(6-2) = A(3-2) + B(-8-1)$

① $4 = A - 9B$

② $(1-2) = A(-2-2) + B(-3-1)$

$-1 = -4A - 4B$

② $1 = 4A + 4B$

From ①, we obtain $A = 9B + 4$.

Substituting $(9B+4)$ for A in EQUATION ②, we obtain

$1 = 4(9B+4) + 4B \Rightarrow 1 = 36B + 16 + 4B$.

$1 = 40B + 16 \Rightarrow 40B = -15 \Rightarrow B = \frac{-15}{40} = -\frac{3}{8}$

$B = -\frac{3}{8}$. $A = 9\left(-\frac{3}{8}\right) + 4 = \frac{-27}{8} + \frac{32}{8} = \frac{5}{8} = A$

So, $(z-2) = \left(\frac{5}{8}\right)(x-2) + \left(-\frac{3}{8}\right)(y-1)$. multiplying by 8:

$8z - 16 = 5(x-2) - 3(y-1) = 5x - 10 - 3y + 3$

$8z - 16 = 5x - 10 - 3y + 3 \Rightarrow \boxed{5x - 3y - 8z = 7 - 16 = -9}$