HW#11 ; Sec 14.1 Sol'ns.

14 ☐ PARTIAL DERIVATIVES

14.1 Functions of Several Variables

1. (a) $f(x,y) = \dfrac{x^2y}{2x-y^2} \;\Rightarrow\; f(1,3) = \dfrac{1^2(3)}{2(1)-3^2} = -\dfrac{3}{7}$ (b) $f(-2,-1) = \dfrac{(-2)^2(-1)}{2(-2)-(-1)^2} = \dfrac{4}{5}$

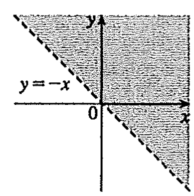
(c) $f(x+h,y) = \dfrac{(x+h)^2y}{2(x+h)-y^2}$ (d) $f(x,x) = \dfrac{x^2x}{2x-x^2} = \dfrac{x^3}{x(2-x)} = \dfrac{x^2}{2-x}$

2. (a) $g(x,y) = x\sin y + y\sin x \;\Rightarrow\; g(\pi,0) = \pi\sin 0 + 0\sin\pi = \pi\cdot 0 + 0\cdot 0 = 0$

(b) $g\left(\frac{\pi}{2},\frac{\pi}{4}\right) = \frac{\pi}{2}\sin\frac{\pi}{4} + \frac{\pi}{4}\sin\frac{\pi}{2} = \frac{\pi}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\pi}{4}(1) = \frac{\pi(\sqrt{2}+1)}{4}$

(c) $g(0,y) = 0\sin y + y\sin 0 = 0 + y\cdot 0 = 0$

(d) $g(x,y+h) = x\sin(y+h) + (y+h)\sin x$

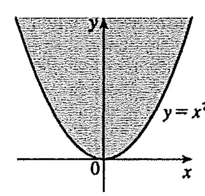


3. (a) $g(x,y) = x^2\ln(x+y) \;\Rightarrow\; g(3,1) = 3^2\ln(3+1) = 9\ln 4$

(b) $\ln(x+y)$ is defined only when $x+y>0 \;\Rightarrow\; y>-x$.

Thus, the domain of g is $\{(x,y)\mid y>-x\}$.

(c) The range of $\ln(x+y)$ is $\mathbb{R}$, so the range of g is $\mathbb{R}$.



4. (a) $h(x,y) = e^{\sqrt{y-x^2}} \;\Rightarrow\; h(-2,5) = e^{\sqrt{5-(-2)^2}} = e^{\sqrt{1}} = e$

(b) $\sqrt{y-x^2}$ is defined only when $y-x^2\ge 0 \;\Rightarrow\; y\ge x^2$.

Thus, the domain of h is $\{(x,y)\mid y\ge x^2\}$.

(c) We know $\sqrt{y-x^2}\ge 0 \;\Rightarrow\; e^{\sqrt{y-x^2}}\ge 1$. Thus, the range of h is $[1,\infty]$.

5. (a) $F(x,y,z) = \sqrt{y} - \sqrt{x-2z} \;\Rightarrow\; F(3,4,1) = \sqrt{4} - \sqrt{3-2(1)} = 2-1 = 1$

(b) $\sqrt{y}$ is defined only when $y\ge 0$. $\sqrt{x-2z}$ is defined only when $x-2z\ge 0 \;\Rightarrow\; z\le\frac{1}{2}x$. Thus, the domain is $\{(x,y,z)\mid x\ge 2z, y\ge 0\}$, which is the set of points on or below the plane $z=\frac{1}{2}x$ and on or to the right of the xz-plane.

6. (a) $f(x,y,z) = \ln\left(z-\sqrt{x^2+y^2}\right) \;\Rightarrow\; f(4,-3,6) = \ln\left(6-\sqrt{4^2+(-3)^2}\right) = \ln\left(6-\sqrt{25}\right) = \ln 1 = 0$

(b) $\ln\left(z-\sqrt{x^2+y^2}\right)$ is defined only when $z-\sqrt{x^2+y^2}>0 \;\Leftrightarrow\; z>\sqrt{x^2+y^2} \;\Rightarrow\; z^2>x^2+y^2$. Thus, the domain is $\{(x,y,z)\mid z>\sqrt{x^2+y^2}\}$, which is the set of points inside the top half of the cone $z^2=x^2+y^2$.

33. The point $(-3, 3)$ lies between the level curves with z-values 50 and 60. Since the point is a little closer to the level curve with $z = 60$, we estimate that $f(-3, 3) \approx 56$. The point $(3, -2)$ appears to be just about halfway between the level curves with z-values 30 and 40, so we estimate $f(3, -2) \approx 35$. The graph rises as we approach the origin, gradually from above, steeply from below.

34. (a) C (Chicago) lies between level curves with pressures 1012 and 1016 mb, and since C appears to be located about one-fourth the distance from the 1012 mb isobar to the 1016 mb isobar, we estimate the pressure at Chicago to be about 1013 mb. N lies very close to a level curve with pressure 1012 mb so we estimate the pressure at Nashville to be approximately 1012 mb. S appears to be just about halfway between level curves with pressures 1008 and 1012 mb, so we estimate the pressure at San Francisco to be about 1010 mb. V lies close to a level curve with pressure 1016 mb but we can't see a level curve to its left so it is more difficult to make an accurate estimate. There are lower pressures to the right of V and V is a short distance to the left of the level curve with pressure 1016 mb, so we might estimate that the pressure at Vancouver is about 1017 mb.

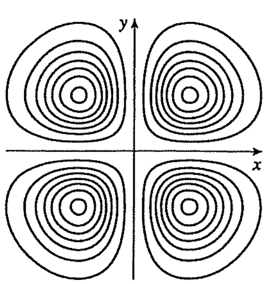
(b) Winds are stronger where the isobars are closer together (see Figure 13), and the level curves are closer near S than at the other locations, so the winds were strongest at San Francisco.

35. The point $(160, 10)$, corresponding to day 160 and a depth of 10 m, lies between the isothermals with temperature values of 8 and $12°C$. Since the point appears to be located about three-fourths the distance from the $8°C$ isothermal to the $12°C$ isothermal, we estimate the temperature at that point to be approximately $11°C$. The point $(180, 5)$ lies between the 16 and $20°C$ isothermals, very close to the $20°C$ level curve, so we estimate the temperature there to be about $19.5°C$.

36. If we start at the origin and move along the x-axis, for example, the z-values of a cone centered at the origin increase at a constant rate, so we would expect its level curves to be equally spaced. A paraboloid with vertex the origin, on the other hand, has z-values which change slowly near the origin and more quickly as we move farther away. Thus, we would expect its level curves near the origin to be spaced more widely apart than those farther from the origin. Therefore contour map I must correspond to the paraboloid, and contour map II the cone.

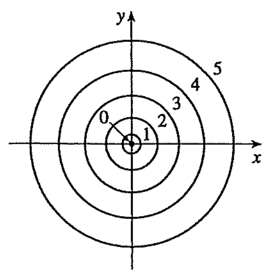
37. Near A, the level curves are very close together, indicating that the terrain is quite steep. At B, the level curves are much farther apart, so we would expect the terrain to be much less steep than near A, perhaps almost flat.

38.

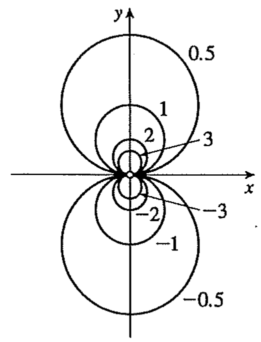


Sec 14.1

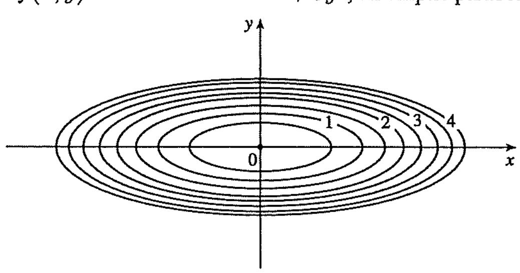
51. The level curves are $\sqrt[3]{x^2+y^2}=k$ or $x^2+y^2=k^3$ $(k \geq 0)$, a family of circles centered at the origin with radius $k^{3/2}$.

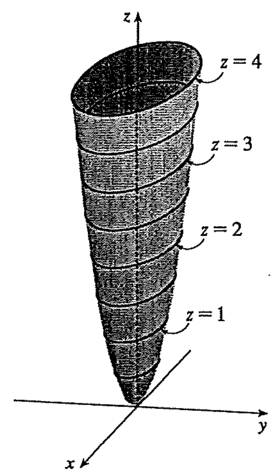


52. For $k \neq 0$ and $(x,y) \neq (0,0)$, $k=\dfrac{y}{x^2+y^2}$ $\Leftrightarrow$ $x^2+y^2-\dfrac{y}{k}=0$ $\Leftrightarrow$ $x^2+\left(y-\frac{1}{2k}\right)^2=\dfrac{1}{4k^2}$, a family of circles with center $\left(0,\frac{1}{2k}\right)$ and radius $\frac{1}{2k}$ (without the origin). If $k=0$, the level curve is the x-axis.



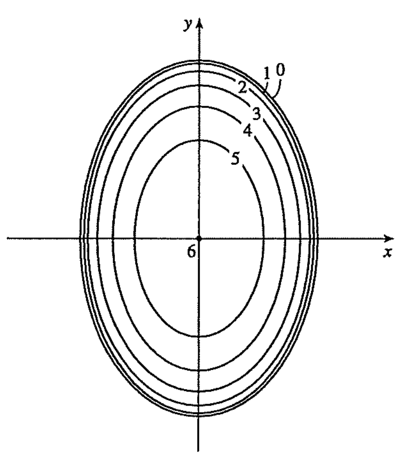
53. The contour map consists of the level curves $k=x^2+9y^2$, a family of ellipses with major axis the x-axis. (Or, if $k=0$, the origin.)
The graph of $f(x,y)$ is the surface $z=x^2+9y^2$, an elliptic paraboloid.



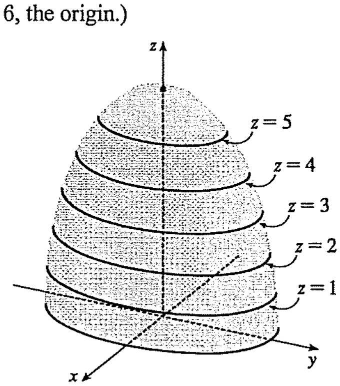


If we visualize lifting each ellipse $k=x^2+9y^2$ of the contour map to the plane $z=k$, we have horizontal traces that indicate the shape of the graph of f.

54.



The contour map consists of the level curves $k=\sqrt{36-9x^2-4y^2}$ $\Rightarrow$ $9x^2+4y^2=36-k^2$, $k \geq 0$, a family of ellipses with major axis the y-axis. (Or, if $k=6$, the origin.)



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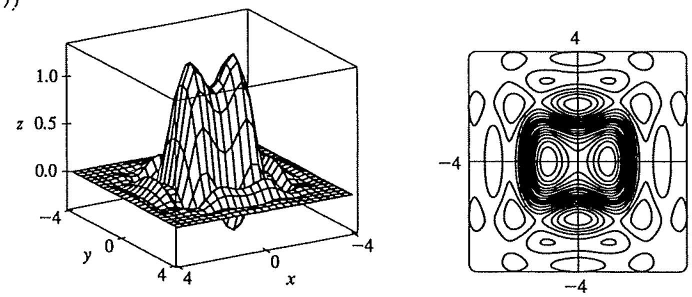
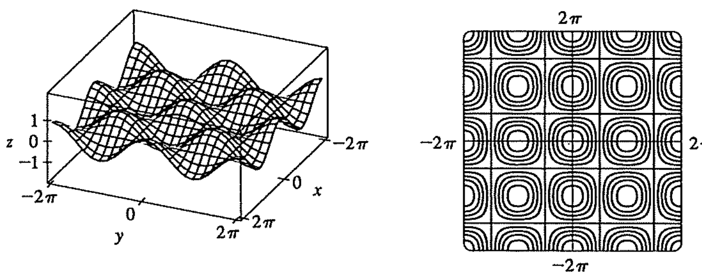
Sec 14.1

59. $f(x,y) = e^{-(x^2+y^2)/3}\left(\sin(x^2) + \cos(y^2)\right)$



60. $f(x,y) = \cos x \cos y$

The traces parallel to either the yz- or xz-plane are cosine curves with amplitudes that vary from 0 to 1.



61. $z = \sin(xy)$ (a) C (b) II

Reasons: This function is periodic in both x and y, and the function is the same when x is interchanged with y, so its graph is symmetric about the plane $y = x$. In addition, the function is 0 along the x- and y-axes. These conditions are satisfied only by C and II.

62. $z = e^x \cos y$ (a) A (b) IV

Reasons: This function is periodic in y but not x, a condition satisfied only by A and IV. Also, note that traces in $x = k$ are cosine curves with amplitude that increases as x increases.

63. $z = \sin(x - y)$ (a) F (b) I

Reasons: This function is periodic in both x and y but is constant along the lines $y = x + k$, a condition satisfied only by F and I.

64. $z = \sin x - \sin y$ (a) E (b) III

Reasons: This function is periodic in both x and y, but unlike the function in Exercise 63, it is not constant along lines such as $y = x + \pi$, so the contour map is III. Also notice that traces in $y = k$ are vertically shifted copies of the sine wave $z = \sin x$, so the graph must be E.

65. $z = (1 - x^2)(1 - y^2)$ (a) B (b) VI

Reasons: This function is 0 along the lines $x = \pm 1$ and $y = \pm 1$. The only contour map in which this could occur is VI. Also note that the trace in the xz-plane is the parabola $z = 1 - x^2$ and the trace in the yz-plane is the parabola $z = 1 - y^2$, so the graph is B.

66. $z = \dfrac{x - y}{1 + x^2 + y^2}$ (a) D (b) V

Reasons: This function is not periodic, ruling out the graphs in A, C, E, and F. Also, the values of z approach 0 as we use points farther from the origin. The only graph that shows this behavior is D, which corresponds to V.