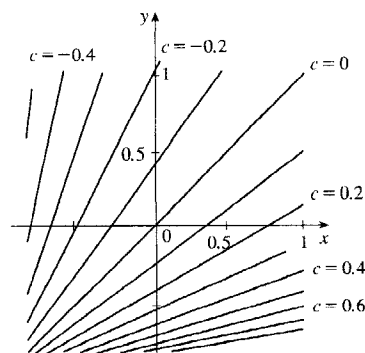


4. The level curves of $f(x, y) = \left(\frac{1+x}{1+y}\right)^{1/3} - 1$ are

$$\left(\frac{1+x}{1+y}\right)^{1/3} - 1 = c \Rightarrow \frac{1+x}{1+y} = (1+c)^3 \Rightarrow$$

$$y = \frac{1+x}{(1+c)^3} - 1.$$

From the level curves, we see that increasing x (from 0) by a small amount has a similar effect on the value of f as decreasing y by a small amount. However, for larger changes, a decrease in y gives greater values of f than a similar increase in x .



14.5 The Chain Rule

1. Find dz/dt using the Chain Rule: $z = x^2y + xy^2$, $x = 3t$, $y = t^2 \Rightarrow$

$$\begin{aligned} \frac{dz}{dt} &= \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = (2xy + y^2)(3) + (x^2 + 2xy)(2t) \\ &= 3[2(3t)(t^2) + (t^2)^2] + 2t[(3t)^2 + 2(3t)(t^2)] \quad [\text{with } x = 3t \text{ and } y = t^2] \\ &= 18t^3 + 3t^4 + 18t^3 + 12t^4 = 36t^3 + 15t^4 \end{aligned}$$

$$\text{Find } dz/dt \text{ by substituting first: } z(x(t), y(t)) = (3t)^2(t^2) + (3t)(t^2)^2 = 9t^4 + 3t^5 \Rightarrow \frac{dz}{dt} = 36t^3 + 15t^4$$

Yes, the two answers agree.

2. Find dz/dt using the Chain Rule: $z = xye^y$, $x = t^2$, $y = 5t \Rightarrow$

$$\begin{aligned} \frac{dz}{dt} &= \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = ye^y(2t) + x(ye^y + e^y)(5) \\ &= (5t)(e^{5t})(2t) + t^2(5te^{5t} + e^{5t})(5) \quad [\text{with } x = t^2 \text{ and } y = 5t] \\ &= 10t^2e^{5t} + 25t^3e^{5t} + 5t^2e^{5t} = 15t^2e^{5t} + 25t^3e^{5t} = 5t^2e^{5t}(3 + 5t) \end{aligned}$$

$$\text{Find } dz/dt \text{ by substituting first: } z(x(t), y(t)) = (t^2)(5t)e^{5t} = 5t^3e^{5t} \Rightarrow \frac{dz}{dt} = 5t^3e^{5t}(5) + 15t^2e^{5t} = 5t^2e^{5t}(3 + 5t)$$

Yes, the two answers agree.

- ③ $z = xy^3 - x^2y$, $x = t^2 + 1$, $y = t^2 - 1 \Rightarrow$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} = (y^3 - 2xy)(2t) + (3xy^2 - x^2)(2t) = 2t(y^3 - 2xy + 3xy^2 - x^2)$$

4. $z = \frac{x-y}{x+2y}$, $x = e^{\pi t}$, $y = e^{-\pi t} \Rightarrow$

$$\begin{aligned} \frac{dz}{dt} &= \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} = \frac{(x+2y)(1) - (x-y)(1)}{(x+2y)^2} (\pi e^{\pi t}) + \frac{(x+2y)(-1) - (x-y)(2)}{(x+2y)^2} (-\pi e^{-\pi t}) \\ &= \frac{3y}{(x+2y)^2} (\pi e^{\pi t}) + \frac{-3x}{(x+2y)^2} (-\pi e^{-\pi t}) = \frac{3\pi}{(x+2y)^2} (ye^{\pi t} + xe^{-\pi t}) \end{aligned}$$

5. $z = \sin x \cos y, \quad x = \sqrt{t}, \quad y = 1/t \Rightarrow$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} = (\cos x \cos y) \left(\frac{1}{2} t^{-1/2} \right) + (-\sin x \sin y) (-t^{-2}) = \frac{1}{2\sqrt{t}} \cos x \cos y + \frac{1}{t^2} \sin x \sin y$$

6. $z = \sqrt{1+xy}, \quad x = \tan t, \quad y = \arctan t \Rightarrow$

$$\begin{aligned} \frac{dz}{dt} &= \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} = \frac{1}{2}(1+xy)^{-1/2}(y) \cdot \sec^2 t + \frac{1}{2}(1+xy)^{-1/2}(x) \cdot \frac{1}{1+t^2} \\ &= \frac{1}{2\sqrt{1+xy}} \left(y \sec^2 t + \frac{x}{1+t^2} \right) \end{aligned}$$

7. $w = xe^{y/z}, \quad x = t^2, \quad y = 1-t, \quad z = 1+2t \Rightarrow$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt} = e^{y/z} \cdot 2t + xe^{y/z} \left(-\frac{1}{z} \right) \cdot (-1) + xe^{y/z} \left(-\frac{y}{z^2} \right) \cdot 2 = e^{y/z} \left(2t - \frac{x}{z} - \frac{2xy}{z^2} \right)$$

8. $w = \ln \sqrt{x^2 + y^2 + z^2} = \frac{1}{2} \ln(x^2 + y^2 + z^2), \quad x = \sin t, \quad y = \cos t, \quad z = \tan t \Rightarrow$

$$\begin{aligned} \frac{dw}{dt} &= \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt} = \frac{1}{2} \cdot \frac{2x}{x^2 + y^2 + z^2} \cdot \cos t + \frac{1}{2} \cdot \frac{2y}{x^2 + y^2 + z^2} \cdot (-\sin t) + \frac{1}{2} \cdot \frac{2z}{x^2 + y^2 + z^2} \cdot \sec^2 t \\ &= \frac{x \cos t - y \sin t + z \sec^2 t}{x^2 + y^2 + z^2} \end{aligned}$$

9. First we find $\partial z / \partial s$ in two ways. $z = x^2 + y^2, \quad x = 2s + 3t, \quad y = s + t \Rightarrow$

$$\begin{aligned} \frac{\partial z}{\partial s} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s} = 2x(2) + 2y(1) \\ &= 2(2s + 3t)(2) + 2(s + t)(1) \quad [\text{with } x = 2s + 3t \text{ and } y = s + t] \\ &= 8s + 12t + 2s + 2t = 10s + 14t \end{aligned}$$

$$z(x(s, t), y(s, t)) = (2s + 3t)^2 + (s + t)^2 \Rightarrow \frac{\partial z}{\partial s} = 2(2s + 3t)(2) + 2(s + t)(1) = 8s + 12t + 2s + 2t = 10s + 14t$$

Yes, the two answers agree. Now we find $\partial z / \partial t$ in two ways.

$$\begin{aligned} \frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} = 2x(3) + 2y(1) \\ &= 2(2s + 3t)(3) + 2(s + t)(1) \quad [\text{with } x = 2s + 3t \text{ and } y = s + t] \\ &= 12s + 18t + 2s + 2t = 14s + 20t \end{aligned}$$

$$z(x(s, t), y(s, t)) = (2s + 3t)^2 + (s + t)^2 \Rightarrow \frac{\partial z}{\partial t} = 2(2s + 3t)(3) + 2(s + t)(1) = 12s + 18t + 2s + 2t = 14s + 20t$$

Yes, the two answers agree.

10. First we find $\partial z / \partial s$ in two ways. $z = x^2 \sin y, \quad x = s^2 t, \quad y = st \Rightarrow$

$$\begin{aligned} \frac{\partial z}{\partial s} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s} = 2x \sin y (2st) + x^2 \cos y (t) \\ &= 2(s^2 t) \sin(st)(2st) + (s^2 t)^2 \cos(st)(t) \quad [\text{with } x = s^2 t \text{ and } y = st] \\ &= 4s^3 t^2 \sin(st) + s^4 t^3 \cos(st) \end{aligned}$$

[continued]

$$z(x(s, t), y(s, t)) = (s^2 t)^2 \sin(st) = s^4 t^2 \sin(st) \Rightarrow$$

$$\frac{\partial z}{\partial s} = s^4 t^2 \cos(st)(t) + 4s^3 t^2 \sin(st) = 4s^3 t^2 \sin(st) + s^4 t^3 \cos(st)$$

Yes, the two answers agree. Now we find $\partial z / \partial t$ in two ways.

$$\begin{aligned} \frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} = 2x \sin y(s^2) + x^2 \cos y(s) \\ &= 2(s^2 t) \sin(st)(s^2) + (s^2 t)^2 \cos(st)(s) \quad [\text{with } x = s^2 t \text{ and } y = st] \\ &= 2s^4 t \sin(st) + s^5 t^2 \cos(st) \end{aligned}$$

$$z(x(s, t), y(s, t)) = (s^2 t)^2 \sin(st) = s^4 t^2 \sin(st) \Rightarrow$$

$$\frac{\partial z}{\partial t} = 2s^4 t \sin(st) + s^4 t^2 \cos(st)(s) = 2s^4 t \sin(st) + s^5 t^2 \cos(st)$$

Yes, the two answers agree.

$$(11) \quad z = (x - y)^5, \quad x = s^2 t, \quad y = st^2 \Rightarrow$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} = 5(x - y)^4(1) \cdot 2st + 5(x - y)^4(-1) \cdot t^2 = 5(x - y)^4(2st - t^2)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t} = 5(x - y)^4(1) \cdot s^2 + 5(x - y)^4(-1) \cdot 2st = 5(x - y)^4(s^2 - 2st)$$

$$12. \quad z = \tan^{-1}(x^2 + y^2), \quad x = s \ln t, \quad y = te^s \Rightarrow$$

$$\begin{aligned} \frac{\partial z}{\partial s} &= \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} = \frac{2x}{1 + (x^2 + y^2)^2} \cdot \ln t + \frac{2y}{1 + (x^2 + y^2)^2} \cdot te^s \\ &= \frac{2}{1 + (x^2 + y^2)^2} (x \ln t + y te^s) \end{aligned}$$

$$\begin{aligned} \frac{\partial z}{\partial t} &= \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t} = \frac{2x}{1 + (x^2 + y^2)^2} \cdot \frac{s}{t} + \frac{2y}{1 + (x^2 + y^2)^2} \cdot e^s \\ &= \frac{2}{1 + (x^2 + y^2)^2} \left(\frac{xs}{t} + ye^s \right) \end{aligned}$$

$$13. \quad z = \ln(3x + 2y), \quad x = s \sin t, \quad y = t \cos s \Rightarrow$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} = \frac{3}{3x + 2y} (\sin t) + \frac{2}{3x + 2y} (-t \sin s) = \frac{3 \sin t - 2t \sin s}{3x + 2y}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t} = \frac{3}{3x + 2y} (s \cos t) + \frac{2}{3x + 2y} (\cos s) = \frac{3s \cos t + 2 \cos s}{3x + 2y}$$

$$(14) \quad z = \sqrt{x} e^{xy}, \quad x = 1 + st, \quad y = s^2 - t^2 \Rightarrow$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial s} = \left(\sqrt{x} \cdot e^{xy}(y) + e^{xy} \cdot \frac{1}{2} x^{-1/2} \right) (t) + \sqrt{x} e^{xy}(x) (2s) = \left(yt\sqrt{x} + \frac{t}{2\sqrt{x}} + 2x^{3/2}s \right) e^{xy}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t} = \left(\sqrt{x} \cdot e^{xy}(y) + e^{xy} \cdot \frac{1}{2} x^{-1/2} \right) (s) + \sqrt{x} e^{xy}(x) (-2t) = \left(ys\sqrt{x} + \frac{s}{2\sqrt{x}} - 2x^{3/2}t \right) e^{xy}$$

15. $z = (\sin \theta)/r$, $r = st$, $\theta = s^2 + t^2 \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial s} = -\frac{\sin \theta}{r^2}(t) + \frac{\cos \theta}{r}(2s) = -\frac{t \sin \theta}{r^2} + \frac{2s \cos \theta}{r}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial t} + \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} = -\frac{\sin \theta}{r^2}(s) + \frac{\cos \theta}{r}(2t) = -\frac{s \sin \theta}{r^2} + \frac{2t \cos \theta}{r}$$

16. $z = \tan(u/v)$, $u = 2s + 3t$, $v = 3s - 2t \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial s} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial s} = \sec^2(u/v)(1/v) \cdot 2 + \sec^2(u/v)(-uv^{-2}) \cdot 3$$

$$= \frac{2}{v} \sec^2\left(\frac{u}{v}\right) - \frac{3u}{v^2} \sec^2\left(\frac{u}{v}\right) = \frac{2v - 3u}{v^2} \sec^2\left(\frac{u}{v}\right)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial t} = \sec^2(u/v)(1/v) \cdot 3 + \sec^2(u/v)(-uv^{-2}) \cdot (-2)$$

$$= \frac{3}{v} \sec^2\left(\frac{u}{v}\right) + \frac{2u}{v^2} \sec^2\left(\frac{u}{v}\right) = \frac{2u + 3v}{v^2} \sec^2\left(\frac{u}{v}\right)$$

17. Let $x = g(t)$ and $y = h(t)$. Then $p(t) = f(x, y)$ and the Chain Rule (2) gives $\frac{dp}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$. When $t = 2$,

$$x = g(2) = 4 \text{ and } y = h(2) = 5, \text{ so } p'(2) = f_x(4, 5)g'(2) + f_y(4, 5)h'(2) = (2)(-3) + (8)(6) = 42.$$

18. $R(s, t) = G(u(s, t), v(s, t)) \Rightarrow \frac{\partial R}{\partial s} = \frac{\partial R}{\partial u} \frac{\partial u}{\partial s} + \frac{\partial R}{\partial v} \frac{\partial v}{\partial s}$ and $\frac{\partial R}{\partial t} = \frac{\partial R}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial R}{\partial v} \frac{\partial v}{\partial t}$ by the

Chain Rule (3). When $s = 1$ and $t = 2$, $u(1, 2) = 5$ and $v(1, 2) = 7$.

$$\text{Thus } R_s(1, 2) = G_u(5, 7)u_s(1, 2) + G_v(5, 7)v_s(1, 2) = (9)(4) + (-2)(2) = 32 \text{ and}$$

$$R_t(1, 2) = G_u(5, 7)u_t(1, 2) + G_v(5, 7)v_t(1, 2) = (9)(-3) + (-2)(6) = -39.$$

19. $g(u, v) = f(x(u, v), y(u, v))$ where $x = e^u + \sin v$, $y = e^u + \cos v \Rightarrow$

$$\frac{\partial x}{\partial u} = e^u, \quad \frac{\partial x}{\partial v} = \cos v, \quad \frac{\partial y}{\partial u} = e^u, \quad \frac{\partial y}{\partial v} = -\sin v. \text{ By the Chain Rule (3), } \frac{\partial g}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u}. \text{ Then}$$

$$g_u(0, 0) = f_x(x(0, 0), y(0, 0))x_u(0, 0) + f_y(x(0, 0), y(0, 0))y_u(0, 0) = f_x(1, 2)(e^0) + f_y(1, 2)(e^0) = 2(1) + 5(1) = 7.$$

$$\text{Similarly, } \frac{\partial g}{\partial v} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial v}. \text{ Then}$$

$$g_v(0, 0) = f_x(x(0, 0), y(0, 0))x_v(0, 0) + f_y(x(0, 0), y(0, 0))y_v(0, 0) = f_x(1, 2)(\cos 0) + f_y(1, 2)(-\sin 0) \\ = 2(1) + 5(0) = 2$$

20. $g(r, s) = f(x(r, s), y(r, s))$ where $x = 2r - s$, $y = s^2 - 4r \Rightarrow \frac{\partial x}{\partial r} = 2$, $\frac{\partial x}{\partial s} = -1$, $\frac{\partial y}{\partial r} = -4$, $\frac{\partial y}{\partial s} = 2s$.

$$\text{By the Chain Rule (3) } \frac{\partial g}{\partial r} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial r}. \text{ Then}$$

$$g_r(1, 2) = f_x(x(1, 2), y(1, 2))x_r(1, 2) + f_y(x(1, 2), y(1, 2))y_r(1, 2) = f_x(0, 0)(2) + f_y(0, 0)(-4) \\ = 4(2) + 8(-4) = -24$$

[continued]

41. $C = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + 0.016D$, so $\frac{\partial C}{\partial T} = 4.6 - 0.11T + 0.00087T^2$ and $\frac{\partial C}{\partial D} = 0.016$.

According to the graph, the diver is experiencing a temperature of approximately 12.5°C at $t = 20$ minutes, so

$$\frac{\partial C}{\partial T} = 4.6 - 0.11(12.5) + 0.00087(12.5)^2 \approx 3.36. \text{ By sketching tangent lines at } t = 20 \text{ to the graphs given, we estimate}$$

$$\frac{dD}{dt} \approx \frac{1}{2} \text{ and } \frac{dT}{dt} \approx -\frac{1}{10}. \text{ Then, by the Chain Rule, } \frac{dC}{dt} = \frac{\partial C}{\partial T} \frac{dT}{dt} + \frac{\partial C}{\partial D} \frac{dD}{dt} \approx (3.36)\left(-\frac{1}{10}\right) + (0.016)\left(\frac{1}{2}\right) \approx -0.33.$$

Thus the speed of sound experienced by the diver is decreasing at a rate of approximately 0.33 m/s per minute.

42. $V = \pi r^2 h/3$, so $\frac{dV}{dt} = \frac{\partial V}{\partial r} \frac{dr}{dt} + \frac{\partial V}{\partial h} \frac{dh}{dt} = \frac{2\pi r h}{3} 1.8 + \frac{\pi r^2}{3} (-2.5) = 20,160\pi - 12,000\pi = 8160\pi \text{ in}^3/\text{s}.$

43. (a) $V = \ell w h$, so by the Chain Rule,

$$\frac{dV}{dt} = \frac{\partial V}{\partial \ell} \frac{d\ell}{dt} + \frac{\partial V}{\partial w} \frac{dw}{dt} + \frac{\partial V}{\partial h} \frac{dh}{dt} = wh \frac{d\ell}{dt} + \ell h \frac{dw}{dt} + \ell w \frac{dh}{dt} = 2 \cdot 2 \cdot 2 + 1 \cdot 2 \cdot 2 + 1 \cdot 2 \cdot (-3) = 6 \text{ m}^3/\text{s}.$$

(b) $S = 2(\ell w + \ell h + wh)$, so by the Chain Rule,

$$\begin{aligned} \frac{dS}{dt} &= \frac{\partial S}{\partial \ell} \frac{d\ell}{dt} + \frac{\partial S}{\partial w} \frac{dw}{dt} + \frac{\partial S}{\partial h} \frac{dh}{dt} = 2(w + h) \frac{d\ell}{dt} + 2(\ell + h) \frac{dw}{dt} + 2(\ell + w) \frac{dh}{dt} \\ &= 2(2 + 2)2 + 2(1 + 2)2 + 2(1 + 2)(-3) = 10 \text{ m}^2/\text{s} \end{aligned}$$

(c) $L^2 = \ell^2 + w^2 + h^2 \Rightarrow 2L \frac{dL}{dt} = 2\ell \frac{d\ell}{dt} + 2w \frac{dw}{dt} + 2h \frac{dh}{dt} = 2(1)(2) + 2(2)(2) + 2(2)(-3) = 0 \Rightarrow$

$$dL/dt = 0 \text{ m/s}.$$

44. $I = \frac{V}{R} \Rightarrow$

$$\frac{dI}{dt} = \frac{\partial I}{\partial V} \frac{dV}{dt} + \frac{\partial I}{\partial R} \frac{dR}{dt} = \frac{1}{R} \frac{dV}{dt} - \frac{V}{R^2} \frac{dR}{dt} = \frac{1}{R} \frac{dV}{dt} - \frac{I}{R} \frac{dR}{dt} = \frac{1}{400}(-0.01) - \frac{0.08}{400}(0.03) = -0.000031 \text{ A/s}$$

45. $\frac{dP}{dt} = 0.05$, $\frac{dT}{dt} = 0.15$, $V = 8.31 \frac{T}{P}$ and $\frac{dV}{dt} = \frac{8.31}{P} \frac{dT}{dt} - 8.31 \frac{T}{P^2} \frac{dP}{dt}$. Thus when $P = 20$ and $T = 320$,

$$\frac{dV}{dt} = 8.31 \left[\frac{0.15}{20} - \frac{(0.05)(320)}{400} \right] \approx -0.27 \text{ L/s}.$$

46. $P = 1.47L^{0.65}K^{0.35}$ and considering P , L , and K as functions of time t we have

$$\frac{dP}{dt} = \frac{\partial P}{\partial L} \frac{dL}{dt} + \frac{\partial P}{\partial K} \frac{dK}{dt} = 1.47(0.65)L^{-0.35}K^{0.35} \frac{dL}{dt} + 1.47(0.35)L^{0.65}K^{-0.65} \frac{dK}{dt}. \text{ We are given}$$

that $\frac{dL}{dt} = -2$ and $\frac{dK}{dt} = 0.5$, so when $L = 30$ and $K = 8$, the rate of change of production $\frac{dP}{dt}$ is

$$1.47(0.65)(30)^{-0.35}(8)^{0.35}(-2) + 1.47(0.35)(30)^{0.65}(8)^{-0.65}(0.5) \approx -0.596. \text{ Thus production at that time}$$

is decreasing at a rate of about \$596,000 per year.