HW #3, Sec 11.2 Solutions

17. For the series $\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n}\right)$,

$$s_n=\sum_{i=1}^{n}\left(\frac{1}{i+2}-\frac{1}{i}\right)$$

$$=\left(\frac{1}{3}-1\right)+\left(\frac{1}{4}-\frac{1}{2}\right)+\left(\frac{1}{5}-\frac{1}{3}\right)+\left(\frac{1}{6}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n-2}\right)+\left(\frac{1}{n+1}-\frac{1}{n-1}\right)+\left(\frac{1}{n+2}-\frac{1}{n}\right)$$

$$=-1-\frac{1}{2}+\frac{1}{n+1}+\frac{1}{n+2} \quad \text{[telescoping series]}$$

Thus, $\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n}\right)=\lim_{n\to\infty}s_n=\lim_{n\to\infty}\left(-1-\frac{1}{2}+\frac{1}{n+1}+\frac{1}{n+2}\right)=-1-\frac{1}{2}=-\frac{3}{2}$. Converges

18. For the series $\sum_{n=4}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$

$$s_n=\sum_{i=4}^{n}\left(\frac{1}{\sqrt{i}}-\frac{1}{\sqrt{i+1}}\right)=\left(\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{5}}\right)+\left(\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{6}}\right)+\cdots+\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)=\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{n+1}}$$

[telescoping series]

Thus, $\sum_{n=4}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)=\lim_{n\to\infty}s_n=\lim_{n\to\infty}\left(\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{n+1}}\right)=\frac{1}{\sqrt{4}}-0=\frac{1}{2}$. Converges

19. For the series $\sum_{n=1}^{\infty}\frac{3}{n(n+3)}$, $s_n=\sum_{i=1}^{n}\frac{3}{i(i+3)}=\sum_{i=1}^{n}\left(\frac{1}{i}-\frac{1}{i+3}\right)$ [using partial fractions]. The latter sum is

$$\left(1-\tfrac{1}{4}\right)+\left(\tfrac{1}{2}-\tfrac{1}{5}\right)+\left(\tfrac{1}{3}-\tfrac{1}{6}\right)+\left(\tfrac{1}{4}-\tfrac{1}{7}\right)+\cdots+\left(\tfrac{1}{n-3}-\tfrac{1}{n}\right)+\left(\tfrac{1}{n-2}-\tfrac{1}{n+1}\right)+\left(\tfrac{1}{n-1}-\tfrac{1}{n+2}\right)+\left(\tfrac{1}{n}-\tfrac{1}{n+3}\right)$$

$$=1+\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{n+1}-\tfrac{1}{n+2}-\tfrac{1}{n+3} \quad \text{[telescoping series]}$$

Thus, $\sum_{n=1}^{\infty}\frac{3}{n(n+3)}=\lim_{n\to\infty}s_n=\lim_{n\to\infty}\left(1+\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{n+1}-\tfrac{1}{n+2}-\tfrac{1}{n+3}\right)=1+\tfrac{1}{2}+\tfrac{1}{3}=\tfrac{11}{6}$. Converges

20. For the series $\sum_{n=1}^{\infty}\ln\frac{n}{n+1}$,

$$s_n=(\ln 1-\ln 2)+(\ln 2-\ln 3)+(\ln 3-\ln 4)+\cdots+[\ln n-\ln(n+1)]=\ln 1-\ln(n+1)=-\ln(n+1)$$

[telescoping series]

Thus, $\lim_{n\to\infty}s_n=-\infty$, so the series is divergent.

21. For the series $\sum_{n=1}^{\infty}\left(e^{1/n}-e^{1/(n+1)}\right)$,

$$s_n=\sum_{i=1}^{n}\left(e^{1/i}-e^{1/(i+1)}\right)=(e^1-e^{1/2})+(e^{1/2}-e^{1/3})+\cdots+\left(e^{1/n}-e^{1/(n+1)}\right)=e-e^{1/(n+1)}$$

[telescoping series]

Thus, $\sum_{n=1}^{\infty}\left(e^{1/n}-e^{1/(n+1)}\right)=\lim_{n\to\infty}s_n=\lim_{n\to\infty}\left(e-e^{1/(n+1)}\right)=e-e^0=e-1$. Converges

22. Using partial fractions, the partial sums of the series $\sum_{n=2}^{\infty} \frac{1}{n^3 - n}$ are

$$s_n = \sum_{i=2}^{n} \frac{1}{i(i-1)(i+1)} = \sum_{i=2}^{n} \left(-\frac{1}{i} + \frac{1/2}{i-1} + \frac{1/2}{i+1} \right) = \frac{1}{2} \sum_{i=2}^{n} \left(\frac{1}{i-1} - \frac{2}{i} + \frac{1}{i+1} \right)$$

$$= \frac{1}{2} \left[\left(\frac{1}{1} - \frac{2}{2} + \frac{1}{3} \right) + \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4} \right) + \left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5} \right) + \left(\frac{1}{4} - \frac{2}{5} + \frac{1}{6} \right) + \cdots \right.$$

$$\left. + \left(\frac{1}{n-3} - \frac{2}{n-2} + \frac{1}{n-1} \right) + \left(\frac{1}{n-2} - \frac{2}{n-1} + \frac{1}{n} \right) + \left(\frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1} \right) \right]$$

Note: In three consecutive expressions in parentheses, the 3rd term in the first expression plus the 2nd term in the second expression plus the 1st term in the third expression sum to 0.

$$= \frac{1}{2} \left(\frac{1}{1} - \frac{2}{2} + \frac{1}{2} + \frac{1}{n} - \frac{2}{n} + \frac{1}{n+1} \right) = \frac{1}{4} - \frac{1}{2n} + \frac{1}{2n+2}$$

Thus, $\sum_{n=2}^{\infty} \frac{1}{n^3 - n} = \lim_{n \to \infty} s_n = \lim_{n \to \infty} \left(\frac{1}{4} - \frac{1}{2n} + \frac{1}{2n+2} \right) = \frac{1}{4}$.

23. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$ is a geometric series with ratio $r = -\frac{4}{3}$. Since $|r| = \frac{4}{3} > 1$, the series diverges.

24. $4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots$ is a geometric series with ratio $\frac{3}{4}$. Since $|r| = \frac{3}{4} < 1$, the series converges to $\frac{a}{1-r} = \frac{4}{1 - 3/4} = 16$.

25. $10 - 2 + 0.4 - 0.08 + \cdots$ is a geometric series with ratio $-\frac{2}{10} = -\frac{1}{5}$. Since $|r| = \frac{1}{5} < 1$, the series converges to

$$\frac{a}{1-r} = \frac{10}{1-(-1/5)} = \frac{10}{6/5} = \frac{50}{6} = \frac{25}{3}.$$

26. $2 + 0.5 + 0.125 + 0.03125 + \cdots$ is a geometric series with ratio $r = \frac{0.5}{2} = \frac{1}{4}$. Since $|r| = \frac{1}{4} < 1$, the series converges to $\frac{a}{1-r} = \frac{2}{1 - 1/4} = \frac{2}{3/4} = \frac{8}{3}$.

27. $\sum_{n=1}^{\infty} 12\,(0.73)^{n-1}$ is a geometric series with first term $a = 12$ and ratio $r = 0.73$. Since $|r| = 0.73 < 1$, the series converges to $\frac{a}{1-r} = \frac{12}{1 - 0.73} = \frac{12}{0.27} = \frac{12(100)}{27} = \frac{400}{9}$.

28. $\sum_{n=1}^{\infty} \frac{5}{\pi^n} = 5 \sum_{n=1}^{\infty} \left(\frac{1}{\pi} \right)^n$. The latter series is geometric with $a = \frac{1}{\pi}$ and ratio $r = \frac{1}{\pi}$. Since $|r| = \frac{1}{\pi} < 1$, it converges to $\frac{1/\pi}{1 - 1/\pi} = \frac{1}{\pi - 1}$. Thus, the given series converges to $5 \left(\frac{1}{\pi - 1} \right) = \frac{5}{\pi - 1}$.

29. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} = \frac{1}{4} \sum_{n=1}^{\infty} \left(-\frac{3}{4} \right)^{n-1}$. The latter series is geometric with $a = 1$ and ratio $r = -\frac{3}{4}$. Since $|r| = \frac{3}{4} < 1$, it converges to $\frac{1}{1 - (-3/4)} = \frac{4}{7}$. Thus, the given series converges to $\left(\frac{1}{4} \right) \left(\frac{4}{7} \right) = \frac{1}{7}$.

30. $\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n} = 3\sum_{n=0}^{\infty} \left(-\frac{3}{2}\right)^n$ is a geometric series with ratio $r = -\frac{3}{2}$. Since $|r| = \frac{3}{2} > 1$, the series diverges.

31. $\sum_{n=1}^{\infty} \frac{e^{2n}}{6^{n-1}} = \sum_{n=1}^{\infty} \frac{(e^2)^n}{6^n 6^{-1}} = 6\sum_{n=1}^{\infty} \left(\frac{e^2}{6}\right)^n$ is a geometric series with ratio $r = \frac{e^2}{6}$. Since $|r| = \frac{e^2}{6}[\approx 1.23] > 1$, the series diverges.

32. $\sum_{n=1}^{\infty} \frac{6 \cdot 2^{2n-1}}{3^n} = \sum_{n=1}^{\infty} \frac{6(2^2)^n \cdot 2^{-1}}{3^n} = 3\sum_{n=1}^{\infty} \left(\frac{4}{3}\right)^n$ is a geometric series with ratio $r = \frac{4}{3}$. Since $|r| = \frac{4}{3} > 1$, the series diverges.

33. $\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \frac{1}{12} + \frac{1}{15} + \cdots = \sum_{n=1}^{\infty} \frac{1}{3n} = \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n}$. This is a constant multiple of the divergent harmonic series, so it diverges.

34. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \cdots = \sum_{n=1}^{\infty} \frac{n}{n+1}$. This series diverges by the Test for Divergence since

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+1/n} = 1 \neq 0.$$

35. $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{625} + \frac{32}{3125} + \cdots = \sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n$. This series is geometric with $a = \frac{2}{5}$ and ratio $r = \frac{2}{5}$. Since $|r| = \frac{2}{5} < 1$, it converges to $\frac{2/5}{1-2/5} = \frac{2}{3}$.

36. $\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \cdots = \left(\frac{1}{3} + \frac{1}{27} + \frac{1}{243} + \cdots\right) + \left(\frac{2}{9} + \frac{2}{81} + \frac{2}{729} + \cdots\right)$, which are both convergent geometric series with sums $\frac{1/3}{1-1/9} = \frac{3}{8}$ and $\frac{2/9}{1-1/9} = \frac{1}{4}$, so the original series converges and its sum is $\frac{3}{8} + \frac{1}{4} = \frac{5}{8}$.

37. $\sum_{n=1}^{\infty} \frac{2+n}{1-2n}$ diverges by the Test for Divergence since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2+n}{1-2n} = \lim_{n\to\infty} \frac{2/n+1}{1/n-2} = -\frac{1}{2} \neq 0$.

38. $\sum_{k=1}^{\infty} \frac{k^2}{k^2-2k+5}$ diverges by the Test for Divergence since $\lim_{k\to\infty} \frac{k^2}{k^2-2k+5} = \lim_{k\to\infty} \frac{1}{1-2/k+5/k^2} = 1 \neq 0$.

39. $\sum_{n=1}^{\infty} 3^{n+1}4^{-n} = \sum_{n=1}^{\infty} \frac{3^n \cdot 3^1}{4^n} = 3\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$. The latter series is geometric with $a = \frac{3}{4}$ and ratio $r = \frac{3}{4}$. Since $|r| = \frac{3}{4} < 1$, it converges to $\frac{3/4}{1-3/4} = 3$. Thus, the given series converges to $3(3) = 9$.

40. $$\sum_{n=1}^{\infty} [(-0.2)^n + (0.6)^{n-1}] = \sum_{n=1}^{\infty}(-0.2)^n + \sum_{n=1}^{\infty}(0.6)^{n-1} \quad \text{[sum of two geometric series]}$$
$$= \frac{-0.2}{1-(-0.2)} + \frac{1}{1-0.6} = -\frac{1}{6} + \frac{5}{2} = \frac{7}{3}$$

50. $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ diverges by the Test for Divergence since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{e^n}{n^2} = \lim_{x\to\infty} \frac{e^x}{x^2} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{e^x}{2} = \infty \neq 0$.

51. (a) Many people would guess that $x < 1$, but note that x consists of an infinite number of 9s.

(b) $x = 0.99999\ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{10{,}000} + \cdots = \sum_{n=1}^{\infty} \frac{9}{10^n}$, which is a geometric series with $a_1 = 0.9$ and $r = 0.1$. Its sum is $\frac{0.9}{1 - 0.1} = \frac{0.9}{0.9} = 1$, that is, $x = 1$.

(c) The number 1 has two decimal representations, $1.00000\ldots$ and $0.99999\ldots$.

(d) Except for 0, all rational numbers that have a terminating decimal representation can be written in more than one way. For example, 0.5 can be written as $0.49999\ldots$ as well as $0.50000\ldots$.

52. $a_1 = 1, a_n = (5 - n)a_{n-1} \quad\Rightarrow\quad a_2 = (5 - 2)a_1 = 3(1) = 3,\; a_3 = (5 - 3)a_2 = 2(3) = 6,\; a_4 = (5 - 4)a_3 = 1(6) = 6$, $a_5 = (5 - 5)a_4 = 0$, and all succeeding terms equal 0. Thus, $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{4} a_n = 1 + 3 + 6 + 6 = 16$.

53. $0.\overline{8} = \frac{8}{10} + \frac{8}{10^2} + \cdots$ is a geometric series with $a = \frac{8}{10}$ and $r = \frac{1}{10}$. It converges to $\frac{a}{1 - r} = \frac{8/10}{1 - 1/10} = \frac{8}{9}$.

54. $0.\overline{46} = \frac{46}{100} + \frac{46}{100^2} + \cdots$ is a geometric series with $a = \frac{46}{100}$ and $r = \frac{1}{100}$. It converges to $\frac{a}{1 - r} = \frac{46/100}{1 - 1/100} = \frac{46}{99}$.

55. $2.\overline{516} = 2 + \frac{516}{10^3} + \frac{516}{10^6} + \cdots$. Now $\frac{516}{10^3} + \frac{516}{10^6} + \cdots$ is a geometric series with $a = \frac{516}{10^3}$ and $r = \frac{1}{10^3}$. It converges to $\frac{a}{1 - r} = \frac{516/10^3}{1 - 1/10^3} = \frac{516/10^3}{999/10^3} = \frac{516}{999}$. Thus, $2.\overline{516} = 2 + \frac{516}{999} = \frac{2514}{999} = \frac{838}{333}$.

56. $10.1\overline{35} = 10.1 + \frac{35}{10^3} + \frac{35}{10^5} + \cdots$. Now $\frac{35}{10^3} + \frac{35}{10^5} + \cdots$ is a geometric series with $a = \frac{35}{10^3}$ and $r = \frac{1}{10^2}$. It converges to $\frac{a}{1 - r} = \frac{35/10^3}{1 - 1/10^2} = \frac{35/10^3}{99/10^2} = \frac{35}{990}$. Thus, $10.1\overline{35} = 10.1 + \frac{35}{990} = \frac{9999 + 35}{990} = \frac{10{,}034}{990} = \frac{5017}{495}$.

57. $1.234\overline{567} = 1.234 + \frac{567}{10^6} + \frac{567}{10^9} + \cdots$. Now $\frac{567}{10^6} + \frac{567}{10^9} + \cdots$ is a geometric series with $a = \frac{567}{10^6}$ and $r = \frac{1}{10^3}$. It converges to $\frac{a}{1 - r} = \frac{567/10^6}{1 - 1/10^3} = \frac{567/10^6}{999/10^3} = \frac{567}{999{,}000} = \frac{21}{37{,}000}$. Thus,

$$1.234\overline{567} = 1.234 + \frac{21}{37{,}000} = \frac{1234}{1000} + \frac{21}{37{,}000} = \frac{45{,}658}{37{,}000} + \frac{21}{37{,}000} = \frac{45{,}679}{37{,}000}.$$

58. $5.\overline{71358} = 5 + \frac{71{,}358}{10^5} + \frac{71{,}358}{10^{10}} + \cdots$. Now $\frac{71{,}358}{10^5} + \frac{71{,}358}{10^{10}} + \cdots$ is a geometric series with $a = \frac{71{,}358}{10^5}$ and $r = \frac{1}{10^5}$. It converges to $\frac{a}{1 - r} = \frac{71{,}358/10^5}{1 - 1/10^5} = \frac{71{,}358/10^5}{99{,}999/10^5} = \frac{71{,}358}{99{,}999} = \frac{23{,}786}{33{,}333}$. Thus,

$$5.\overline{71358} = 5 + \frac{23{,}786}{33{,}333} = \frac{166{,}665}{33{,}333} + \frac{23{,}786}{33{,}333} = \frac{190{,}451}{33{,}333}.$$

59. $\sum_{n=1}^{\infty}(-5)^n x^n = \sum_{n=1}^{\infty}(-5x)^n$ is a geometric series with $r = -5x$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$

$|-5x| < 1$ $\Leftrightarrow$ $|x| < \frac{1}{5}$, that is, $-\frac{1}{5} < x < \frac{1}{5}$. In that case, the sum of the series is $\dfrac{a}{1-r} = \dfrac{-5x}{1-(-5x)} = \dfrac{-5x}{1+5x}$.

60. $\sum_{n=1}^{\infty}(x+2)^n$ is a geometric series with $r = x+2$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$ $|x+2| < 1$ $\Leftrightarrow$

$-1 < x+2 < 1$ $\Leftrightarrow$ $-3 < x < -1$. In that case, the sum of the series is $\dfrac{a}{1-r} = \dfrac{x+2}{1-(x+2)} = \dfrac{x+2}{-x-1}$.

61. $\sum_{n=0}^{\infty}\dfrac{(x-2)^n}{3^n} = \sum_{n=0}^{\infty}\left(\dfrac{x-2}{3}\right)^n$ is a geometric series with $r = \dfrac{x-2}{3}$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$

$\left|\dfrac{x-2}{3}\right| < 1$ $\Leftrightarrow$ $-1 < \dfrac{x-2}{3} < 1$ $\Leftrightarrow$ $-3 < x-2 < 3$ $\Leftrightarrow$ $-1 < x < 5$. In that case, the sum of the series is

$$\frac{a}{1-r} = \frac{1}{1-\frac{x-2}{3}} = \frac{1}{\frac{3-(x-2)}{3}} = \frac{3}{5-x}.$$

62. $\sum_{n=0}^{\infty}(-4)^n(x-5)^n = \sum_{n=0}^{\infty}[-4(x-5)]^n$ is a geometric series with $r = -4(x-5)$, so the series converges $\Leftrightarrow$

$|r| < 1$ $\Leftrightarrow$ $|-4(x-5)| < 1$ $\Leftrightarrow$ $|x-5| < \frac{1}{4}$ $\Leftrightarrow$ $-\frac{1}{4} < x-5 < \frac{1}{4}$ $\Leftrightarrow$ $\frac{19}{4} < x < \frac{21}{4}$. In that case, the sum of the series is $\dfrac{a}{1-r} = \dfrac{1}{1-[-4(x-5)]} = \dfrac{1}{4x-19}$.

63. $\sum_{n=0}^{\infty}\dfrac{2^n}{x^n} = \sum_{n=0}^{\infty}\left(\dfrac{2}{x}\right)^n$ is a geometric series with $r = \dfrac{2}{x}$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$ $\left|\dfrac{2}{x}\right| < 1$ $\Leftrightarrow$

$2 < |x|$ $\Leftrightarrow$ $x > 2$ or $x < -2$. In that case, the sum of the series is $\dfrac{a}{1-r} = \dfrac{1}{1-2/x} = \dfrac{x}{x-2}$.

64. $\sum_{n=0}^{\infty}\dfrac{x^n}{2^n} = \sum_{n=0}^{\infty}\left(\dfrac{x}{2}\right)^n$ is a geometric series with $r = \dfrac{x}{2}$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$ $\left|\dfrac{x}{2}\right| < 1$ $\Leftrightarrow$

$-1 < \dfrac{x}{2} < 1$ $\Leftrightarrow$ $-2 < x < 2$. In that case, the sum of the series is $\dfrac{a}{1-r} = \dfrac{1}{1-\frac{x}{2}} = \dfrac{2}{2-x}$.

65. $\sum_{n=0}^{\infty}e^{nx} = \sum_{n=0}^{\infty}(e^x)^n$ is a geometric series with $r = e^x$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$ $|e^x| < 1$ $\Leftrightarrow$

$-1 < e^x < 1$ $\Leftrightarrow$ $0 < e^x < 1$ $\Leftrightarrow$ $x < 0$. In that case, the sum of the series is $\dfrac{a}{1-r} = \dfrac{1}{1-e^x}$.

66. $\sum_{n=0}^{\infty}\dfrac{\sin^n x}{3^n} = \sum_{n=0}^{\infty}\left(\dfrac{\sin x}{3}\right)^n$ is a geometric series with $r = \dfrac{\sin x}{3}$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$

$\left|\dfrac{\sin x}{3}\right| < 1$ $\Leftrightarrow$ $|\sin x| < 3$, which is true for all x. Thus, the sum of the series is $\dfrac{a}{1-r} = \dfrac{1}{1-(\sin x)/3} = \dfrac{3}{3-\sin x}$.

67. After defining f, We use `convert(f,parfrac);` in Maple or `Apart` in Mathematica to find that the general term is

$\dfrac{3n^2+3n+1}{(n^2+n)^3} = \dfrac{1}{n^3} - \dfrac{1}{(n+1)^3}$. So the nth partial sum is

$$s_n = \sum_{k=1}^{n}\left(\frac{1}{k^3} - \frac{1}{(k+1)^3}\right) = \left(1 - \frac{1}{2^3}\right) + \left(\frac{1}{2^3} - \frac{1}{3^3}\right) + \cdots + \left(\frac{1}{n^3} - \frac{1}{(n+1)^3}\right) = 1 - \frac{1}{(n+1)^3}$$