

HW #3, Sec 7.8 solutions

$$5. \int_1^{\infty} 2x^{-3} dx = \lim_{t \rightarrow \infty} \int_1^t 2x^{-3} dx = \lim_{t \rightarrow \infty} \left[-x^{-2} \right]_1^t = \lim_{t \rightarrow \infty} \left[-t^{-2} + 1 \right] = 0 + 1 = 1. \quad \text{Convergent}$$

$$6. \int_{-\infty}^{-1} \frac{1}{\sqrt[3]{x}} dx = \lim_{t \rightarrow -\infty} \int_t^{-1} x^{-1/3} dx = \lim_{t \rightarrow -\infty} \left[\frac{3}{2} x^{2/3} \right]_t^{-1} = \lim_{t \rightarrow -\infty} \left[\frac{3}{2} - \frac{3}{2} t^{2/3} \right] = -\infty. \quad \text{Divergent}$$

$$7. \int_0^{\infty} e^{-2x} dx = \lim_{t \rightarrow \infty} \int_0^t e^{-2x} dx = \lim_{t \rightarrow \infty} \left[-\frac{1}{2} e^{-2x} \right]_0^t = \lim_{t \rightarrow \infty} \left[-\frac{1}{2} e^{-2t} + \frac{1}{2} \right] = 0 + \frac{1}{2} = \frac{1}{2}. \quad \text{Convergent}$$

$$8. \int_1^{\infty} \left(\frac{1}{3} \right)^x dx = \lim_{t \rightarrow \infty} \int_1^t 3^{-x} dx = \lim_{t \rightarrow \infty} \left[-\frac{3^{-x}}{\ln 3} \right]_1^t = \lim_{t \rightarrow \infty} \left[-\frac{3^{-t}}{\ln 3} + \frac{3^{-1}}{\ln 3} \right] = 0 + \frac{1}{3 \ln 3} = \frac{1}{\ln 27}. \quad \text{Convergent}$$

$$9. \int_{-2}^{\infty} \frac{1}{x+4} dx = \lim_{t \rightarrow \infty} \int_{-2}^t \frac{1}{x+4} dx = \lim_{t \rightarrow \infty} \left[\ln |x+4| \right]_{-2}^t = \lim_{t \rightarrow \infty} \left[\ln |t+4| - \ln 2 \right] = \infty \text{ since } \lim_{t \rightarrow \infty} \ln |t+4| = \infty. \\ \text{Divergent}$$

$$\textcircled{10}. \int_1^{\infty} \frac{1}{x^2+4} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{1}{x^2+4} dx = \lim_{t \rightarrow \infty} \left[\frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) \right]_1^t = \lim_{t \rightarrow \infty} \left[\frac{1}{2} \tan^{-1} \left(\frac{t}{2} \right) - \frac{1}{2} \tan^{-1} \left(\frac{1}{2} \right) \right] \\ = \frac{\pi}{4} - \frac{1}{2} \tan^{-1} \left(\frac{1}{2} \right). \quad \text{Convergent}$$

$$11. \int_3^{\infty} \frac{1}{(x-2)^{3/2}} dx = \lim_{t \rightarrow \infty} \int_3^t (x-2)^{-3/2} dx = \lim_{t \rightarrow \infty} \left[-2(x-2)^{-1/2} \right]_3^t \quad [u = x-2, du = dx] \\ = \lim_{t \rightarrow \infty} \left(\frac{-2}{\sqrt{t-2}} + \frac{2}{\sqrt{1}} \right) = 0 + 2 = 2. \quad \text{Convergent}$$

$$12. \int_0^{\infty} \frac{1}{\sqrt[4]{1+x}} dx = \lim_{t \rightarrow \infty} \int_0^t (1+x)^{-1/4} dx = \lim_{t \rightarrow \infty} \left[\frac{4}{3} (1+x)^{3/4} \right]_0^t \quad [u = 1+x, du = dx] \\ = \lim_{t \rightarrow \infty} \left[\frac{4}{3} (1+t)^{3/4} - \frac{4}{3} \right] = \infty. \quad \text{Divergent}$$

$$\textcircled{13}. \int_{-\infty}^0 \frac{x}{(x^2+1)^3} dx = \lim_{t \rightarrow -\infty} \int_t^0 \frac{x}{(x^2+1)^3} dx = \lim_{t \rightarrow -\infty} \left[-\frac{1}{4} (x^2+1)^{-2} \right]_t^0 = \lim_{t \rightarrow -\infty} \left[-\frac{1}{4(x^2+1)^2} \right]_t^0 \\ = \lim_{t \rightarrow -\infty} \left[-\frac{1}{4} + \frac{1}{4(t^2+1)^2} \right] = -\frac{1}{4} + 0 = -\frac{1}{4}. \quad \text{Convergent}$$

$$14. \int_{-\infty}^{-3} \frac{x}{4-x^2} dx = \lim_{t \rightarrow -\infty} \int_t^{-3} \frac{x}{4-x^2} dx = \lim_{t \rightarrow -\infty} \left[-\frac{1}{2} \ln |4-x^2| \right]_t^{-3} \\ = \lim_{t \rightarrow -\infty} \left[-\frac{1}{2} \ln 5 + \frac{1}{2} \ln |4-t^2| \right] = \infty \text{ since } \lim_{t \rightarrow -\infty} \ln |4-t^2| = \infty. \quad \text{Divergent}$$

$$15. \int_1^{\infty} \frac{x^2+x+1}{x^4} dx = \lim_{t \rightarrow \infty} \int_1^t (x^{-2} + x^{-3} + x^{-4}) dx \\ = \lim_{t \rightarrow \infty} \left[-x^{-1} - \frac{1}{2} x^{-2} - \frac{1}{3} x^{-3} \right]_1^t = \lim_{t \rightarrow \infty} \left[-\frac{1}{t} - \frac{1}{2t^2} - \frac{1}{3t^3} \right]_1^t \\ = \lim_{t \rightarrow \infty} \left[\left(-\frac{1}{t} - \frac{1}{2t^2} - \frac{1}{3t^3} \right) - \left(-1 - \frac{1}{2} - \frac{1}{3} \right) \right] = 0 + \frac{11}{6} = \frac{11}{6}. \quad \text{Convergent}$$

$$16. \int_2^{\infty} \frac{x}{\sqrt{x^2-1}} dx = \lim_{t \rightarrow \infty} \int_2^t \frac{x}{\sqrt{x^2-1}} dx = \lim_{t \rightarrow \infty} \left[\sqrt{x^2-1} \right]_2^t = \lim_{t \rightarrow \infty} \left[\sqrt{t^2-1} - \sqrt{3} \right] = \infty. \quad \text{Divergent}$$

Sec 7.8

$$17. \int_0^{\infty} \frac{e^x}{(1+e^x)^2} dx = \lim_{t \rightarrow \infty} \int_0^t \frac{e^x}{(1+e^x)^2} dx = \lim_{t \rightarrow \infty} \left[-(1+e^x)^{-1} \right]_0^t = \lim_{t \rightarrow \infty} \left[-\frac{1}{1+e^x} \right]_0^t$$

$$= \lim_{t \rightarrow \infty} \left[-\frac{1}{1+e^t} + \frac{1}{2} \right] = 0 + \frac{1}{2} = \frac{1}{2}. \quad \text{Convergent}$$

$$18. I = \int_{-\infty}^{-1} \frac{x^2+x}{x^3} dx = \int_{-\infty}^{-1} \frac{1}{x} dx + \int_{-\infty}^{-1} \frac{1}{x^2} dx = I_1 + I_2.$$

$$\text{Now, } \int_{-\infty}^{-1} \frac{1}{x} dx = \lim_{t \rightarrow -\infty} \int_t^{-1} \frac{1}{x} dx = \lim_{t \rightarrow -\infty} \left[\ln|x| \right]_t^{-1} = \lim_{t \rightarrow -\infty} \left[\ln 1 - \ln|t| \right] = -\infty.$$

Since I_1 is divergent, I is divergent. Divergent

$$19. \int_{-\infty}^{\infty} x e^{-x^2} dx = \int_{-\infty}^0 x e^{-x^2} dx + \int_0^{\infty} x e^{-x^2} dx.$$

$$\int_{-\infty}^0 x e^{-x^2} dx = \lim_{t \rightarrow -\infty} \left(-\frac{1}{2} \right) \left[e^{-x^2} \right]_t^0 = \lim_{t \rightarrow -\infty} \left(-\frac{1}{2} \right) (1 - e^{-t^2}) = -\frac{1}{2} \cdot (1 - 0) = -\frac{1}{2}, \text{ and}$$

$$\int_0^{\infty} x e^{-x^2} dx = \lim_{t \rightarrow \infty} \left(-\frac{1}{2} \right) \left[e^{-x^2} \right]_0^t = \lim_{t \rightarrow \infty} \left(-\frac{1}{2} \right) (e^{-t^2} - 1) = -\frac{1}{2} \cdot (0 - 1) = \frac{1}{2}.$$

Therefore, $\int_{-\infty}^{\infty} x e^{-x^2} dx = -\frac{1}{2} + \frac{1}{2} = 0$. Convergent

$$20. I = \int_{-\infty}^{\infty} \frac{x}{x^2+1} dx = \int_{-\infty}^0 \frac{x}{x^2+1} dx + \int_0^{\infty} \frac{x}{x^2+1} dx = I_1 + I_2, \text{ but}$$

$$I_2 = \lim_{t \rightarrow \infty} \int_0^t \frac{x}{x^2+1} dx = \lim_{t \rightarrow \infty} \int_1^{t^2+1} \frac{1/2}{u} du \quad \left[\begin{array}{l} u = x^2+1, \\ du = 2x dx \end{array} \right] = \frac{1}{2} \lim_{t \rightarrow \infty} \left[\ln|u| \right]_1^{t^2+1}$$

$$= \frac{1}{2} \lim_{t \rightarrow \infty} \left[\ln|t^2+1| - 0 \right] = \infty$$

Since I_2 is divergent, I is divergent, and there is no need to evaluate I_1 . Divergent

21. $I = \int_{-\infty}^{\infty} \cos 2t dt = \int_{-\infty}^0 \cos 2t dt + \int_0^{\infty} \cos 2t dt = I_1 + I_2$, but $I_1 = \lim_{s \rightarrow -\infty} \left[\frac{1}{2} \sin 2t \right]_s^0 = \lim_{s \rightarrow -\infty} \left(-\frac{1}{2} \sin 2s \right)$, and this limit does not exist. Since I_1 is divergent, I is divergent, and there is no need to evaluate I_2 . Divergent

$$22. \int_1^{\infty} \frac{e^{-1/x}}{x^2} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{e^{-1/x}}{x^2} dx = \lim_{t \rightarrow \infty} \left[e^{-1/x} \right]_1^t = \lim_{t \rightarrow \infty} (e^{-1/t} - e^{-1}) = 1 - \frac{1}{e}. \quad \text{Convergent}$$

$$23. \int_0^{\infty} \sin^2 \alpha d\alpha = \lim_{t \rightarrow \infty} \int_0^t \frac{1}{2} (1 - \cos 2\alpha) d\alpha = \lim_{t \rightarrow \infty} \left[\frac{1}{2} \left(\alpha - \frac{1}{2} \sin 2\alpha \right) \right]_0^t = \lim_{t \rightarrow \infty} \left[\frac{1}{2} \left(t - \frac{1}{2} \sin 2t \right) - 0 \right] = \infty.$$

Divergent

$$24. \int_0^{\infty} \sin \theta e^{\cos \theta} d\theta = \lim_{t \rightarrow \infty} \int_0^t \sin \theta e^{\cos \theta} d\theta = \lim_{t \rightarrow \infty} \left[-e^{\cos \theta} \right]_0^t = \lim_{t \rightarrow \infty} (-e^{\cos t} + e)$$

This limit does not exist since $\cos t$ oscillates in value between -1 and 1 , so $e^{\cos t}$ oscillates in value between e^{-1} and e^1 . Divergent

$$25. \int_1^{\infty} \frac{1}{x^2+x} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{1}{x(x+1)} dx = \lim_{t \rightarrow \infty} \int_1^t \left(\frac{1}{x} - \frac{1}{x+1} \right) dx \quad [\text{partial fractions}]$$

$$= \lim_{t \rightarrow \infty} \left[\ln|x| - \ln|x+1| \right]_1^t = \lim_{t \rightarrow \infty} \left[\ln \left| \frac{x}{x+1} \right| \right]_1^t = \lim_{t \rightarrow \infty} \left(\ln \frac{t}{t+1} - \ln \frac{1}{2} \right) = 0 - \ln \frac{1}{2} = \ln 2.$$

Convergent

$$\begin{aligned}
 26. \int_2^{\infty} \frac{dv}{v^2 + 2v - 3} &= \lim_{t \rightarrow \infty} \int_2^t \frac{dv}{(v+3)(v-1)} = \lim_{t \rightarrow \infty} \int_2^t \left(\frac{-\frac{1}{4}}{v+3} + \frac{\frac{1}{4}}{v-1} \right) dv = \lim_{t \rightarrow \infty} \left[-\frac{1}{4} \ln |v+3| + \frac{1}{4} \ln |v-1| \right]_2^t \\
 &= \frac{1}{4} \lim_{t \rightarrow \infty} \left[\ln \frac{v-1}{v+3} \right]_2^t = \frac{1}{4} \lim_{t \rightarrow \infty} \left(\ln \frac{t-1}{t+3} - \ln \frac{1}{5} \right) = \frac{1}{4} (0 + \ln 5) = \frac{1}{4} \ln 5. \quad \text{Convergent}
 \end{aligned}$$

$$\begin{aligned}
 27. \int_{-\infty}^0 z e^{2z} dz &= \lim_{t \rightarrow -\infty} \int_t^0 z e^{2z} dz = \lim_{t \rightarrow -\infty} \left[\frac{1}{2} z e^{2z} - \frac{1}{4} e^{2z} \right]_t^0 \quad \left[\begin{array}{l} \text{integration by parts with} \\ u = z, dv = e^{2z} dz \end{array} \right] \\
 &= \lim_{t \rightarrow -\infty} \left[\left(0 - \frac{1}{4} \right) - \left(\frac{1}{2} t e^{2t} - \frac{1}{4} e^{2t} \right) \right] = -\frac{1}{4} - 0 + 0 \quad [\text{by l'Hospital's Rule}] = -\frac{1}{4}. \quad \text{Convergent}
 \end{aligned}$$

$$\begin{aligned}
 28. \int_2^{\infty} y e^{-3y} dy &= \lim_{t \rightarrow \infty} \int_2^t y e^{-3y} dy = \lim_{t \rightarrow \infty} \left[-\frac{1}{3} y e^{-3y} - \frac{1}{9} e^{-3y} \right]_2^t \quad \left[\begin{array}{l} \text{integration by parts with} \\ u = y, dv = e^{-3y} dy \end{array} \right] \\
 &= \lim_{t \rightarrow \infty} \left[\left(-\frac{1}{3} t e^{-3t} - \frac{1}{9} e^{-3t} \right) - \left(-\frac{2}{3} e^{-6} - \frac{1}{9} e^{-6} \right) \right] = 0 - 0 + \frac{7}{9} e^{-6} \quad [\text{by l'Hospital's Rule}] = \frac{7}{9} e^{-6}.
 \end{aligned}$$

Convergent

$$29. \int_1^{\infty} \frac{\ln x}{x} dx = \lim_{t \rightarrow \infty} \left[\frac{(\ln x)^2}{2} \right]_1^t \quad \left[\begin{array}{l} \text{by substitution with} \\ u = \ln x, du = dx/x \end{array} \right] = \lim_{t \rightarrow \infty} \frac{(\ln t)^2}{2} = \infty. \quad \text{Divergent}$$

$$\begin{aligned}
 30. \int_1^{\infty} \frac{\ln x}{x^2} dx &= \lim_{t \rightarrow \infty} \int_1^t \frac{\ln x}{x^2} dx = \lim_{t \rightarrow \infty} \left[-\frac{\ln x}{x} - \frac{1}{x} \right]_1^t \quad \left[\begin{array}{l} \text{integration by parts with} \\ u = \ln x, dv = (1/x^2) dx \end{array} \right] \\
 &= \lim_{t \rightarrow \infty} \left(-\frac{\ln t}{t} - \frac{1}{t} + 1 \right) \stackrel{H}{=} \lim_{t \rightarrow \infty} \left(-\frac{1/t}{1} \right) - \lim_{t \rightarrow \infty} \frac{1}{t} + \lim_{t \rightarrow \infty} 1 = 0 - 0 + 1 = 1. \quad \text{Convergent}
 \end{aligned}$$

$$\begin{aligned}
 31. \int_{-\infty}^0 \frac{z}{z^4 + 4} dz &= \lim_{t \rightarrow -\infty} \int_t^0 \frac{z}{z^4 + 4} dz = \lim_{t \rightarrow -\infty} \frac{1}{2} \left[\frac{1}{2} \tan^{-1} \left(\frac{z^2}{2} \right) \right]_t^0 \quad \left[\begin{array}{l} u = z^2, \\ du = 2z dz \end{array} \right] \\
 &= \lim_{t \rightarrow -\infty} \left[0 - \frac{1}{4} \tan^{-1} \left(\frac{t^2}{2} \right) \right] = -\frac{1}{4} \left(\frac{\pi}{2} \right) = -\frac{\pi}{8}. \quad \text{Convergent}
 \end{aligned}$$

$$\begin{aligned}
 32. \int_e^{\infty} \frac{1}{x(\ln x)^2} dx &= \lim_{t \rightarrow \infty} \int_e^t \frac{1}{x(\ln x)^2} dx = \lim_{t \rightarrow \infty} \left[-\frac{1}{\ln x} \right]_e^t \quad \left[\begin{array}{l} u = \ln x, \\ du = (1/x) dx \end{array} \right] \\
 &= \lim_{t \rightarrow \infty} \left(-\frac{1}{\ln t} + 1 \right) = 0 + 1 = 1. \quad \text{Convergent}
 \end{aligned}$$

$$\begin{aligned}
 33. \int_0^{\infty} e^{-\sqrt{y}} dy &= \lim_{t \rightarrow \infty} \int_0^t e^{-\sqrt{y}} dy = \lim_{t \rightarrow \infty} \int_0^{\sqrt{t}} e^{-x} (2x dx) \quad \left[\begin{array}{l} x = \sqrt{y}, \\ dx = 1/(2\sqrt{y}) dy \end{array} \right] \\
 &= \lim_{t \rightarrow \infty} \left\{ [-2x e^{-x}]_0^{\sqrt{t}} + \int_0^{\sqrt{t}} 2e^{-x} dx \right\} \quad \left[\begin{array}{l} u = 2x, \quad dv = e^{-x} dx \\ du = 2 dx, \quad v = -e^{-x} \end{array} \right] \\
 &= \lim_{t \rightarrow \infty} \left(-2\sqrt{t} e^{-\sqrt{t}} + [-2e^{-x}]_0^{\sqrt{t}} \right) = \lim_{t \rightarrow \infty} \left(\frac{-2\sqrt{t}}{e^{\sqrt{t}}} - \frac{2}{e^{\sqrt{t}}} + 2 \right) = 0 - 0 + 2 = 2.
 \end{aligned}$$

Convergent

$$\text{Note: } \lim_{t \rightarrow \infty} \frac{\sqrt{t}}{e^{\sqrt{t}}} \stackrel{H}{=} \lim_{t \rightarrow \infty} \frac{2\sqrt{t}}{2\sqrt{t} e^{\sqrt{t}}} = \lim_{t \rightarrow \infty} \frac{1}{e^{\sqrt{t}}} = 0$$

Sec 7.8

$$\begin{aligned} 34. \int_1^{\infty} \frac{dx}{\sqrt{x} + x\sqrt{x}} &= \lim_{t \rightarrow \infty} \int_1^t \frac{dx}{\sqrt{x}(1+x)} = \lim_{t \rightarrow \infty} \int_1^{\sqrt{t}} \frac{1}{u(1+u^2)} (2u \, du) \quad \left[\begin{array}{l} u = \sqrt{x}, \\ du = 1/(2\sqrt{x}) \, dx \end{array} \right] \\ &= \lim_{t \rightarrow \infty} \int_1^{\sqrt{t}} \frac{2}{1+u^2} \, du = \lim_{t \rightarrow \infty} [2 \tan^{-1} u]_1^{\sqrt{t}} = \lim_{t \rightarrow \infty} 2(\tan^{-1} \sqrt{t} - \tan^{-1} 1) \\ &= 2\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\pi}{2}. \quad \text{Convergent} \end{aligned}$$

$$35. \int_0^1 \frac{1}{x} \, dx = \lim_{t \rightarrow 0^+} \int_t^1 \frac{1}{x} \, dx = \lim_{t \rightarrow 0^+} [\ln |x|]_t^1 = \lim_{t \rightarrow 0^+} (-\ln t) = \infty. \quad \text{Divergent}$$

$$\begin{aligned} 36. \int_0^5 \frac{1}{\sqrt[3]{5-x}} \, dx &= \lim_{t \rightarrow 5^-} \int_0^t (5-x)^{-1/3} \, dx = \lim_{t \rightarrow 5^-} \left[-\frac{3}{2}(5-x)^{2/3} \right]_0^t = \lim_{t \rightarrow 5^-} \left\{ -\frac{3}{2}[(5-t)^{2/3} - 5^{2/3}] \right\} \\ &= \frac{3}{2} 5^{2/3}. \quad \text{Convergent} \end{aligned}$$

$$\begin{aligned} 37. \int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}} &= \lim_{t \rightarrow -2^+} \int_t^{14} (x+2)^{-1/4} \, dx = \lim_{t \rightarrow -2^+} \left[\frac{4}{3}(x+2)^{3/4} \right]_t^{14} = \frac{4}{3} \lim_{t \rightarrow -2^+} [16^{3/4} - (t+2)^{3/4}] \\ &= \frac{4}{3}(8-0) = \frac{32}{3}. \quad \text{Convergent} \end{aligned}$$

$$\begin{aligned} 38. \int_{-1}^2 \frac{x}{(x+1)^2} \, dx &= \lim_{t \rightarrow -1^+} \int_t^2 \frac{x}{(x+1)^2} \, dx = \lim_{t \rightarrow -1^+} \int_t^2 \left[\frac{1}{x+1} - \frac{1}{(x+1)^2} \right] \, dx \quad [\text{partial fractions}] \\ &= \lim_{t \rightarrow -1^+} \left[\ln |x+1| + \frac{1}{x+1} \right]_t^2 = \lim_{t \rightarrow -1^+} \left[\ln 3 + \frac{1}{3} - \left(\ln(t+1) + \frac{1}{t+1} \right) \right] = -\infty. \quad \text{Divergent} \end{aligned}$$

Note: To justify the last step, $\lim_{t \rightarrow -1^+} \left[\ln(t+1) + \frac{1}{t+1} \right] = \lim_{x \rightarrow 0^+} \left(\ln x + \frac{1}{x} \right) \quad \left[\begin{array}{l} \text{substitute} \\ x \text{ for } t+1 \end{array} \right] = \lim_{x \rightarrow 0^+} \frac{x \ln x + 1}{x} = \infty$

since $\lim_{x \rightarrow 0^+} (x \ln x) = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2} = \lim_{x \rightarrow 0^+} (-x) = 0$.

$$39. \int_{-2}^3 \frac{1}{x^4} \, dx = \int_{-2}^0 \frac{dx}{x^4} + \int_0^3 \frac{dx}{x^4}, \text{ but } \int_{-2}^0 \frac{dx}{x^4} = \lim_{t \rightarrow 0^-} \left[-\frac{x^{-3}}{3} \right]_{-2}^t = \lim_{t \rightarrow 0^-} \left[-\frac{1}{3t^3} - \frac{1}{24} \right] = \infty. \quad \text{Divergent}$$

$$40. \int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t \rightarrow 1^-} \int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t \rightarrow 1^-} [\sin^{-1} x]_0^t = \lim_{t \rightarrow 1^-} \sin^{-1} t = \frac{\pi}{2}. \quad \text{Convergent}$$

$$41. \text{ There is an infinite discontinuity at } x = 1. \quad \int_0^9 \frac{1}{\sqrt[3]{x-1}} \, dx = \int_0^1 (x-1)^{-1/3} \, dx + \int_1^9 (x-1)^{-1/3} \, dx.$$

$$\text{Here } \int_0^1 (x-1)^{-1/3} \, dx = \lim_{t \rightarrow 1^-} \int_0^t (x-1)^{-1/3} \, dx = \lim_{t \rightarrow 1^-} \left[\frac{3}{2}(x-1)^{2/3} \right]_0^t = \lim_{t \rightarrow 1^-} \left[\frac{3}{2}(t-1)^{2/3} - \frac{3}{2} \right] = -\frac{3}{2}$$

$$\text{and } \int_1^9 (x-1)^{-1/3} \, dx = \lim_{t \rightarrow 1^+} \int_t^9 (x-1)^{-1/3} \, dx = \lim_{t \rightarrow 1^+} \left[\frac{3}{2}(x-1)^{2/3} \right]_t^9 = \lim_{t \rightarrow 1^+} \left[6 - \frac{3}{2}(t-1)^{2/3} \right] = 6. \text{ Thus,}$$

$$\int_0^9 \frac{1}{\sqrt[3]{x-1}} \, dx = -\frac{3}{2} + 6 = \frac{9}{2}. \quad \text{Convergent}$$

$$42. \text{ There is an infinite discontinuity at } w = 2.$$

$$\int_0^2 \frac{w}{w-2} \, dw = \lim_{t \rightarrow 2^-} \int_0^t \left(1 + \frac{2}{w-2} \right) \, dw = \lim_{t \rightarrow 2^-} [w + 2 \ln |w-2|]_0^t = \lim_{t \rightarrow 2^-} (t + 2 \ln |t-2| - 2 \ln 2) = -\infty, \text{ so}$$

$$\int_0^2 \frac{w}{w-2} \, dw \text{ diverges, and hence, } \int_0^5 \frac{w}{w-2} \, dw \text{ diverges.} \quad \text{Divergent}$$