Hw #5; Section 11.5 Solutions

3. $-\dfrac{2}{5} + \dfrac{4}{6} - \dfrac{6}{7} + \dfrac{8}{8} - \dfrac{10}{9} + \cdots = \displaystyle\sum_{n=1}^{\infty}(-1)^n\dfrac{2n}{n+4}$. Now $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty}\dfrac{2n}{n+4} = \lim_{n\to\infty}\dfrac{2}{1+4/n} = \dfrac{2}{1} \neq 0$. Since

$\displaystyle\lim_{n\to\infty} a_n \neq 0$ (in fact the limit does not exist), the series diverges by the Test for Divergence.

4. $\dfrac{1}{\ln 3} - \dfrac{1}{\ln 4} + \dfrac{1}{\ln 5} - \dfrac{1}{\ln 6} + \dfrac{1}{\ln 7} - \cdots = \displaystyle\sum_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{\ln(n+2)}$. Now $b_n = \dfrac{1}{\ln(n+2)} > 0$, $\{b_n\}$ is decreasing, and $\displaystyle\lim_{n\to\infty} b_n = 0$,

so the series converges by the Alternating Series Test.

5. $\displaystyle\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{3+5n} = \sum_{n=1}^{\infty}(-1)^{n-1}b_n$. Now $b_n = \dfrac{1}{3+5n} > 0$, $\{b_n\}$ is decreasing, and $\displaystyle\lim_{n\to\infty} b_n = 0$, so the series

converges by the Alternating Series Test.

⑥. $\displaystyle\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty}\dfrac{(-1)^{n+1}}{\sqrt{n+1}} = \sum_{n=0}^{\infty}(-1)^{n+1}b_n$. Now $b_n = \dfrac{1}{\sqrt{n+1}} > 0$, $\{b_n\}$ is decreasing, and $\displaystyle\lim_{n\to\infty} b_n = 0$, so the series

converges by the Alternating Series Test.

7. $\displaystyle\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(-1)^n\dfrac{3n-1}{2n+1} = \sum_{n=1}^{\infty}(-1)^n b_n$. Now $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty}\dfrac{3-1/n}{2+1/n} = \dfrac{3}{2} \neq 0$. Since $\displaystyle\lim_{n\to\infty} a_n \neq 0$

(in fact the limit does not exist), the series diverges by the Test for Divergence.

8. $\displaystyle\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(-1)^n\dfrac{n^2}{n^2+n+1} = \sum_{n=1}^{\infty}(-1)^n b_n$. Now $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty}\dfrac{n^2}{n^2+n+1} = \lim_{n\to\infty}\dfrac{1}{1+1/n+1/n^2} = 1 \neq 0$.

Since $\displaystyle\lim_{n\to\infty} a_n \neq 0$, the series diverges by the Test for Divergence.

9. $\displaystyle\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(-1)^n e^{-n} = \sum_{n=1}^{\infty}(-1)^n b_n$. Now $b_n = \dfrac{1}{e^n} > 0$, $\{b_n\}$ is decreasing, and $\displaystyle\lim_{n\to\infty} b_n = 0$, so the series converges

by the Alternating Series Test.

⑩. $b_n = \dfrac{\sqrt{n}}{2n+3} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 2$ since

$$\left(\dfrac{\sqrt{x}}{2x+3}\right)' = \dfrac{(2x+3)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2)}{(2x+3)^2} = \dfrac{\frac{1}{2}x^{-1/2}[(2x+3) - 4x]}{(2x+3)^2} = \dfrac{3-2x}{2\sqrt{x}\,(2x+3)^2} < 0 \text{ for } x > \tfrac{3}{2}.$$

Also, $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty}\dfrac{\sqrt{n}/\sqrt{n}}{(2n+3)/\sqrt{n}} = \lim_{n\to\infty}\dfrac{1}{2\sqrt{n}+3/\sqrt{n}} = 0$. Thus, the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n\dfrac{\sqrt{n}}{2n+3}$ converges by the

Alternating Series Test.

11. $b_n = \dfrac{n^2}{n^3+4} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 2$ since

$$\left(\dfrac{x^2}{x^3+4}\right)' = \dfrac{(x^3+4)(2x) - x^2(3x^2)}{(x^3+4)^2} = \dfrac{x(2x^3+8-3x^3)}{(x^3+4)^2} = \dfrac{x(8-x^3)}{(x^3+4)^2} < 0 \text{ for } x > 2. \text{ Also,}$$

$\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty}\dfrac{1/n}{1+4/n^3} = 0$. Thus, the series $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\dfrac{n^2}{n^3+4}$ converges by the Alternating Series Test.

12. $b_n = \dfrac{n}{2^n} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 2$ since $\left(\dfrac{x}{2^x}\right)' = \dfrac{2^x - x\,2^x \ln 2}{(2^x)^2} = \dfrac{1 - x\ln 2}{2^x} < 0$ for $x > \dfrac{1}{\ln 2} \approx 1.4$.

Also, $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{n}{2^n} \overset{\text{H}}{=} \lim\limits_{n\to\infty} \dfrac{1}{2^n \ln 2} = 0$. Thus, the series $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n}{2^n}$ converges by the Alternating Series Test.

13. $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} e^{2/n} = e^0 = 1$, so $\lim\limits_{n\to\infty} (-1)^{n-1} e^{2/n}$ does not exist. Thus, the series $\sum\limits_{n=1}^{\infty} (-1)^{n-1} e^{2/n}$ diverges by the

Test for Divergence.

14. $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \arctan n = \tfrac{\pi}{2}$, so $\lim\limits_{n\to\infty} (-1)^{n-1} \arctan n$ does not exist. Thus, the series $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \arctan n$ diverges

by the Test for Divergence.

15. $a_n = \dfrac{\sin\left(n + \tfrac{1}{2}\right)\pi}{1 + \sqrt{n}} = \dfrac{(-1)^n}{1 + \sqrt{n}}$. Now $b_n = \dfrac{1}{1 + \sqrt{n}} > 0$ for $n \geq 0$, $\{b_n\}$ is decreasing, and $\lim\limits_{n\to\infty} b_n = 0$, so the series

$\sum\limits_{n=0}^{\infty} \dfrac{\sin\left(n + \tfrac{1}{2}\right)\pi}{1 + \sqrt{n}}$ converges by the Alternating Series Test.

16. $a_n = \dfrac{n \cos n\pi}{2^n} = (-1)^n \dfrac{n}{2^n} = (-1)^n b_n$. $\{b_n\}$ is decreasing for $n \geq 2$ since

$(x\,2^{-x})' = x(-2^{-x} \ln 2) + 2^{-x} = 2^{-x}(1 - x\ln 2) < 0$ for $x > \dfrac{1}{\ln 2}$ $[\approx 1.4]$. Also, $\lim\limits_{n\to\infty} b_n = 0$ since

$\lim\limits_{x\to\infty} \dfrac{x}{2^x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{1}{2^x \ln 2} = 0$. Thus, the series $\sum\limits_{n=1}^{\infty} \dfrac{n \cos n\pi}{2^n}$ converges by the Alternating Series Test.

17. $\sum\limits_{n=1}^{\infty} (-1)^n \sin\dfrac{\pi}{n}$. $b_n = \sin\dfrac{\pi}{n} > 0$ for $n \geq 2$ and $\sin\dfrac{\pi}{n} \geq \sin\dfrac{\pi}{n+1}$, and $\lim\limits_{n\to\infty} \sin\dfrac{\pi}{n} = \sin 0 = 0$, so the series converges by

the Alternating Series Test.

18. $\sum\limits_{n=1}^{\infty} (-1)^n \cos\dfrac{\pi}{n}$. $\lim\limits_{n\to\infty} \cos\dfrac{\pi}{n} = \cos(0) = 1$, so $\lim\limits_{n\to\infty} (-1)^n \cos\dfrac{\pi}{n}$ does not exist and the series diverges by the Test for

Divergence.

19. $b_n = \dfrac{n^2}{5^n} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 2$ since

$\left(\dfrac{x^2}{5^x}\right)' = \dfrac{5^x \cdot 2x - x^2\,5^x \ln 5}{(5^x)^2} = \dfrac{x\,5^x(2 - x\ln 5)}{(5^x)^2} = \dfrac{x(2 - x\ln 5)}{5^x} < 0$ for $x > \dfrac{2}{\ln 5} \approx 1.2$. Also,

$\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{n^2}{5^n} \overset{\text{H}}{=} \lim\limits_{n\to\infty} \dfrac{2n}{5^n \ln 5} \overset{\text{H}}{=} \lim\limits_{n\to\infty} \dfrac{2}{5^n (\ln 5)^2} = 0$. Thus, the series $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n^2}{5^n}$ converges by the Alternating

Series Test.

20. $b_n = \dfrac{\sqrt{n+1} - \sqrt{n}}{1} \cdot \dfrac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \dfrac{(n+1) - n}{\sqrt{n+1} + \sqrt{n}} = \dfrac{1}{\sqrt{n+1} + \sqrt{n}} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing and

$\lim\limits_{n\to\infty} b_n = 0$, so the series $\sum\limits_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right)$ converges by the Alternating Series Test.

21. (a) A series $\sum a_n$ is called absolutely convergent if the series of absolute values $\sum |a_n|$ is convergent. If a series is absolutely convergent, then it is convergent.

(b) A series $\sum a_n$ is called conditionally convergent if it is convergent but not absolutely convergent; that is, if $\sum a_n$ converges, but $\sum |a_n|$ diverges.

(c) Suppose the series of positive terms $\sum_{n=1}^{\infty} b_n$ converges. Then $\sum |(-1)^n b_n| = \sum |b_n| = \sum b_n$ also converges, so $\sum_{n=1}^{\infty} (-1)^n b_n$ is absolutely convergent (and therefore convergent).

22. $\sum\limits_{n=1}^{\infty} |a_n| = \sum\limits_{n=1}^{\infty} \left| \dfrac{(-1)^n}{n^4} \right| = \sum\limits_{n=1}^{\infty} \dfrac{1}{n^4}$. Since $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^4}$ is a convergent p-series $[p = 4 > 1]$, the series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^4}$ is absolutely convergent.

23. $b_n = \dfrac{1}{\sqrt[3]{n^2}} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 1$, and $\lim\limits_{n \to \infty} \dfrac{1}{\sqrt[3]{n^2}} = 0$, so the series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{\sqrt[3]{n^2}}$ converges by the

Alternating Series Test. Also, observe that $\sum\limits_{n=1}^{\infty} \left| \dfrac{(-1)^{n-1}}{\sqrt[3]{n^2}} \right| = \sum\limits_{n=1}^{\infty} \dfrac{1}{n^{2/3}}$ is divergent since it is a p-series with $p = \frac{2}{3} \leq 1$.

Thus, the series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{\sqrt[3]{n^2}}$ is conditionally convergent.

24. Since $\lim\limits_{n \to \infty} \dfrac{n^2}{n^2 + 1} = \lim\limits_{n \to \infty} \dfrac{1}{1 + 1/n^2} = \dfrac{1}{1 + 0} = 1 \neq 0$ and $\lim\limits_{n \to \infty} (-1)^{n+1}$ does not exist, $\lim\limits_{n \to \infty} (-1)^{n+1} \dfrac{n^2}{n^2 + 1}$ does not

exist, so the series $\sum\limits_{n=0}^{\infty} (-1)^{n+1} \dfrac{n^2}{n^2 + 1}$ diverges by the Test for Divergence.

25. $b_n = \dfrac{1}{5n + 1} > 0$ for $n \geq 1$, $\{b_n\}$ is decreasing for $n \geq 1$, and $\lim\limits_{n \to \infty} b_n = 0$, so $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{5n + 1}$ converges by the Alternating

Series Test. To determine absolute convergence, choose $a_n = \dfrac{1}{n}$ to get

$\lim\limits_{n \to \infty} \dfrac{a_n}{b_n} = \lim\limits_{n \to \infty} \dfrac{1/n}{1/(5n + 1)} = \lim\limits_{n \to \infty} \dfrac{5n + 1}{n} = 5 > 0$, so $\sum\limits_{n=1}^{\infty} \dfrac{1}{5n + 1}$ diverges by the Limit Comparison Test with the

harmonic series. Thus, the series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{5n + 1}$ is conditionally convergent.

26. $\sum\limits_{n=1}^{\infty} \dfrac{-n}{n^2 + 1} = -\sum\limits_{n=1}^{\infty} \dfrac{n}{n^2 + 1}$. Use the Limit Comparison Test with $a_n = \dfrac{n}{n^2 + 1}$ and $b_n = \dfrac{1}{n}$:

$$\lim\limits_{n \to \infty} \dfrac{a_n}{b_n} = \lim\limits_{n \to \infty} \dfrac{n/(n^2 + 1)}{1/n} = \lim\limits_{n \to \infty} \dfrac{n^2}{n^2 + 1} = \lim\limits_{n \to \infty} \dfrac{1}{1 + 1/n^2} = \dfrac{1}{1 + 0} = 1 > 0$$

Since $\sum\limits_{n=1}^{\infty} \dfrac{1}{n}$ is a divergent p-series $[p = 1 \leq 1]$, the series $\sum\limits_{n=1}^{\infty} \dfrac{n}{n^2 + 1}$ also diverges, and hence, the negative of this series,

$\sum\limits_{n=1}^{\infty} \dfrac{-n}{n^2 + 1}$, diverges.

27. $\sum\limits_{n=1}^{\infty} |a_n| = \sum\limits_{n=1}^{\infty} \left| \dfrac{(-1)^n}{n^2+1} \right| = \sum\limits_{n=1}^{\infty} \dfrac{1}{n^2+1}$. Since $\dfrac{1}{n^2+1} < \dfrac{1}{n^2}$ and $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^2}$ is a convergent p-series $[p=2>1]$, the series

$\sum\limits_{n=1}^{\infty} \dfrac{1}{n^2+1}$ is convergent by the Direct Comparison Test. Thus, $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^2+1}$ is absolutely convergent.

28. $0 < \left| \dfrac{\sin n}{2^n} \right| < \dfrac{1}{2^n}$ for $n \geq 1$ and $\sum\limits_{n=1}^{\infty} \dfrac{1}{2^n}$ is a convergent geometric series $[r = \frac{1}{2} < 1]$, so $\sum\limits_{n=1}^{\infty} \left| \dfrac{\sin n}{2^n} \right|$ converges by direct

comparison and the series $\sum\limits_{n=1}^{\infty} \dfrac{\sin n}{2^n}$ is absolutely convergent.

29. $0 < \left| \dfrac{1+2\sin n}{n^3} \right| < \dfrac{3}{n^3}$ for $n \geq 1$ and $3\sum\limits_{n=1}^{\infty} \dfrac{1}{n^3}$ is a constant times a convergent p-series $[p=3>1]$, so $\sum\limits_{n=1}^{\infty} \left| \dfrac{1+2\sin n}{n^3} \right|$

converges by direct comparison and the series $\sum\limits_{n=1}^{\infty} \dfrac{1+2\sin n}{n^3}$ is absolutely convergent.

30. $b_n = \dfrac{n}{n^2+4} > 0$ for $n \geq 1$, $\{b_n\}$ is decreasing for $n \geq 2$, and $\lim\limits_{n\to\infty} b_n = 0$, so $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{n}{n^2+4}$ converges by the

Alternating Series Test. To determine absolute convergence, choose $a_n = \dfrac{1}{n}$ to get

$\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = \lim\limits_{n\to\infty} \dfrac{1/n}{n/(n^2+4)} = \lim\limits_{n\to\infty} \dfrac{n^2+4}{n^2} = \lim\limits_{n\to\infty} \dfrac{1+4/n^2}{1} = 1 > 0$, so $\sum\limits_{n=1}^{\infty} \dfrac{n}{n^2+4}$ diverges by the Limit

Comparison Test with the harmonic series. Thus, the series $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{n}{n^2+4}$ is conditionally convergent.

31. $\sum\limits_{n=2}^{\infty} \dfrac{(-1)^n}{\ln n}$ converges by the Alternating Series Test since $\lim\limits_{n\to\infty} \dfrac{1}{\ln n} = 0$ and $\left\{ \dfrac{1}{\ln n} \right\}$ is decreasing. Now $\ln n < n$, so

$\dfrac{1}{\ln n} > \dfrac{1}{n}$, and since $\sum\limits_{n=2}^{\infty} \dfrac{1}{n}$ is the divergent (partial) harmonic series, $\sum\limits_{n=2}^{\infty} \dfrac{1}{\ln n}$ diverges by the Direct Comparison Test. Thus,

$\sum\limits_{n=2}^{\infty} \dfrac{(-1)^n}{\ln n}$ is conditionally convergent.

32. $b_n = \dfrac{n}{\sqrt{n^3+2}} > 0$ for $n \geq 1$, $\{b_n\}$ is decreasing for $n \geq 2$, and $\lim\limits_{n\to\infty} b_n = 0$, so $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n}{\sqrt{n^3+2}}$ converges by

the Alternating Series Test. To determine absolute convergence, choose $a_n = \dfrac{1}{\sqrt{n}}$ to get

$\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = \lim\limits_{n\to\infty} \left(\dfrac{1}{\sqrt{n}} \cdot \dfrac{\sqrt{n^3+2}}{n} \right) = \lim\limits_{n\to\infty} \dfrac{\sqrt{n^3+2}}{\sqrt{n^3}} = \lim\limits_{n\to\infty} \sqrt{1 + \dfrac{2}{n^3}} = 1 > 0$, so $\sum\limits_{n=1}^{\infty} \dfrac{n}{\sqrt{n^3+2}}$ diverges by limit

comparision with the divergent p-series $\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt{n}}$ $[p = \frac{1}{2} \leq 1]$. Thus, $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n}{\sqrt{n^3+2}}$ is conditionally convergent.

33. $a_n = \dfrac{\cos n\pi}{3n+2} = (-1)^n \dfrac{1}{3n+2} = (-1)^n b_n$. $\{b_n\}$ is decreasing for $n \geq 1$, and $\lim\limits_{n\to\infty} b_n = 0$, so $\sum\limits_{n=1}^{\infty} \dfrac{\cos n\pi}{3n+2}$ converges by

the Alternating Series Test. To determine absolute convergence, use the Limit Comparison Test with $a_n = \dfrac{1}{n}$:

[continued]

37. The series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^6}$ satisfies (i) of the Alternating Series Test because $\dfrac{1}{(n+1)^6} < \dfrac{1}{n^6}$ and (ii) $\lim\limits_{n\to\infty} \dfrac{1}{n^6} = 0$, so the

series is convergent. Now $b_5 = \dfrac{1}{5^6} = 0.000064 > 0.00005$ and $b_6 = \dfrac{1}{6^6} \approx 0.00002 < 0.00005$, so by the Alternating Series

Estimation Theorem, $n = 5$. (That is, since the 6th term is less than the desired error, we need to add the first 5 terms to get the

sum to the desired accuracy.)

38. The series $\sum\limits_{n=1}^{\infty} \dfrac{(-\frac{1}{3})^n}{n} = \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{1}{n3^n}$ satisfies (i) of the Alternating Series Test because $\dfrac{1}{(n+1)3^{n+1}} < \dfrac{1}{n3^n}$ and

(ii) $\lim\limits_{n\to\infty} \dfrac{1}{n3^n} = 0$, so the series is convergent. Now $b_5 = \dfrac{1}{5 \cdot 3^5} \approx 0.0008 > 0.0005$ and $b_6 = \dfrac{1}{6 \cdot 3^6} \approx 0.0002 < 0.0005$,

so by the Alternating Series Estimation Theorem, $n = 5$. (That is, since the 6th term is less than the desired error, we need to

add the first 5 terms to get the sum to the desired accuracy.)

39. The series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{n^2 2^n}$ satisfies (i) of the Alternating Series Test because $\dfrac{1}{(n+1)^2 2^{n+1}} < \dfrac{1}{n^2 2^n}$ and (ii) $\lim\limits_{n\to\infty} \dfrac{1}{n^2 2^n} = 0$,

so the series is convergent. Now $b_5 = \dfrac{1}{5^2 2^5} = 0.00125 > 0.0005$ and $b_6 = \dfrac{1}{6^2 2^6} \approx 0.0004 < 0.0005$, so by the Alternating

Series Estimation Theorem, $n = 5$. (That is, since the 6th term is less than the desired error, we need to add the first 5 terms to

get the sum to the desired accuracy.)

40. The series $\sum\limits_{n=1}^{\infty} \left(-\dfrac{1}{n}\right)^n = \sum\limits_{n=1}^{\infty} (-1)^n \dfrac{1}{n^n}$ satisfies (i) of the Alternating Series Test because $\dfrac{1}{(n+1)^{n+1}} < \dfrac{1}{n^n}$ and

(ii) $\lim\limits_{n\to\infty} \dfrac{1}{n^n} = 0$, so the series is convergent. Now $b_5 = \dfrac{1}{5^5} = 0.00032 > 0.00005$ and $b_6 = \dfrac{1}{6^6} \approx 0.00002 < 0.00005$, so

by the Alternating Series Estimation Theorem, $n = 5$. (That is, since the 6th term is less than the desired error, we need to add

the first 5 terms to get the sum to the desired accuracy.)

41. $b_4 = \dfrac{1}{8!} = \dfrac{1}{40{,}320} \approx 0.000\,025$, so

$$\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{(2n)!} \approx s_3 = \sum\limits_{n=1}^{3} \dfrac{(-1)^n}{(2n)!} = -\dfrac{1}{2} + \dfrac{1}{24} - \dfrac{1}{720} \approx -0.459\,722$$

Adding b_4 to s_3 does not change the fourth decimal place of s_3, so by the Alternating Series Estimation Theorem, the sum of

the series, correct to four decimal places, is -0.4597.

42. $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^6} \approx s_9 = \dfrac{1}{1^6} - \dfrac{1}{2^6} + \dfrac{1}{3^6} - \dfrac{1}{4^6} + \dfrac{1}{5^6} - \dfrac{1}{6^6} + \dfrac{1}{7^6} - \dfrac{1}{8^6} + \dfrac{1}{9^6} \approx 0.985\,552$. Subtracting $b_{10} = 1/10^6$ from s_9

does not change the fourth decimal place of s_9, so by the Alternating Series Estimation Theorem, the sum of the series, correct

to four decimal places, is 0.9856.

43. $\sum\limits_{n=1}^{\infty} (-1)^n n e^{-2n} \approx s_5 = -\dfrac{1}{e^2} + \dfrac{2}{e^4} - \dfrac{3}{e^6} + \dfrac{4}{e^8} - \dfrac{5}{e^{10}} \approx -0.105\,025$. Adding $b_6 = 6/e^{12} \approx 0.000\,037$ to s_5 does not

change the fourth decimal place of s_5, so by the Alternating Series Estimation Theorem, the sum of the series, correct to four

decimal places, is -0.1050.