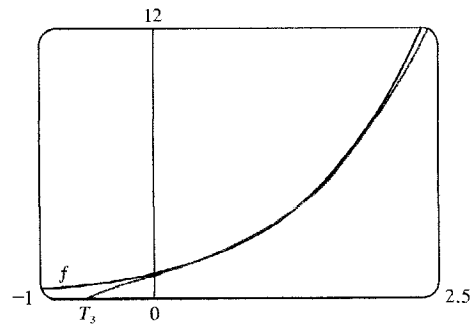


HW #6, SEC 11.11 SOLUTIONS

3.

n	$f^{(n)}(x)$	$f^{(n)}(1)$
0	e^x	e
1	e^x	e
2	e^x	e
3	e^x	e

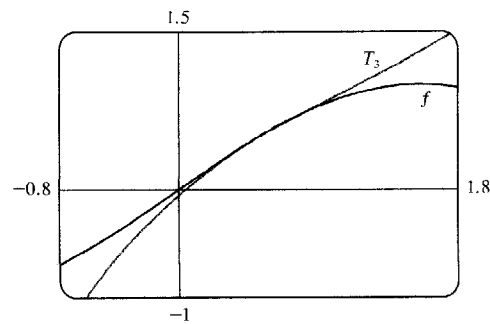
$$\begin{aligned}
 T_3(x) &= \sum_{n=0}^3 \frac{f^{(n)}(1)}{n!} (x-1)^n \\
 &= \frac{e}{0!} (x-1)^0 + \frac{e}{1!} (x-1)^1 + \frac{e}{2!} (x-1)^2 + \frac{e}{3!} (x-1)^3 \\
 &= e + e(x-1) + \frac{1}{2}e(x-1)^2 + \frac{1}{6}e(x-1)^3
 \end{aligned}$$



4.

n	$f^{(n)}(x)$	$f^{(n)}(\pi/6)$
0	$\sin x$	$1/2$
1	$\cos x$	$\sqrt{3}/2$
2	$-\sin x$	$-1/2$
3	$-\cos x$	$-\sqrt{3}/2$

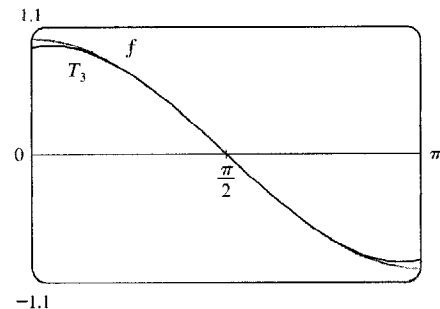
$$\begin{aligned}
 T_3(x) &= \sum_{n=0}^3 \frac{f^{(n)}(\pi/6)}{n!} \left(x - \frac{\pi}{6}\right)^n \\
 &= \frac{1/2}{0!} \left(x - \frac{\pi}{6}\right)^0 + \frac{\sqrt{3}/2}{1!} \left(x - \frac{\pi}{6}\right)^1 - \frac{1/2}{2!} \left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}/2}{3!} \left(x - \frac{\pi}{6}\right)^3 \\
 &= \frac{1}{2} + \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{6}\right) - \frac{1}{4} \left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{12} \left(x - \frac{\pi}{6}\right)^3
 \end{aligned}$$



5.

n	$f^{(n)}(x)$	$f^{(n)}(\pi/2)$
0	$\cos x$	0
1	$-\sin x$	-1
2	$-\cos x$	0
3	$\sin x$	1

$$\begin{aligned}
 T_3(x) &= \sum_{n=0}^3 \frac{f^{(n)}(\pi/2)}{n!} \left(x - \frac{\pi}{2}\right)^n \\
 &= -\left(x - \frac{\pi}{2}\right) + \frac{1}{6} \left(x - \frac{\pi}{2}\right)^3
 \end{aligned}$$



6.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$e^{-x} \sin x$	0
1	$e^{-x} (\cos x - \sin x)$	1
2	$-2e^{-x} \cos x$	-2
3	$2e^{-x} (\cos x + \sin x)$	2

$$T_3(x) = \sum_{n=0}^3 \frac{f^{(n)}(0)}{n!} x^n = x - x^2 + \frac{1}{3}x^3$$

