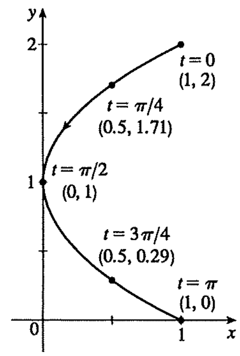
HW # 7 , SEC 10.1 SOLUTIONS Sec 10.1

6. $x = \cos^2 t, \quad y = 1 + \cos t, \quad 0 \le t \le \pi$

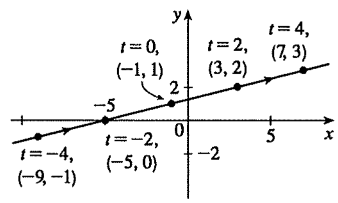
t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
x	1	0.5	0	0.5	1
y	2	1.707	1	0.293	0



7. $x = 2t - 1, \quad y = \frac{1}{2}t + 1$

(a)

t	-4	-2	0	2	4
x	-9	-5	-1	3	7
y	-1	0	1	2	3



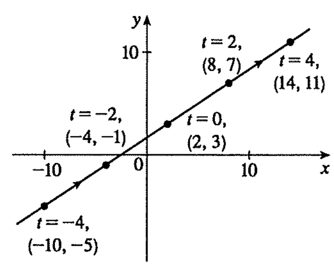
(b) $x = 2t - 1 \;\Rightarrow\; 2t = x + 1 \;\Rightarrow\; t = \frac{1}{2}x + \frac{1}{2}$, so

$y = \frac{1}{2}t + 1 = \frac{1}{2}\left(\frac{1}{2}x + \frac{1}{2}\right) + 1 = \frac{1}{4}x + \frac{1}{4} + 1 \;\Rightarrow\; y = \frac{1}{4}x + \frac{5}{4}$

8. $x = 3t + 2, \quad y = 2t + 3$

(a)

t	-4	-2	0	2	4
x	-10	-4	2	8	14
y	-5	-1	3	7	11



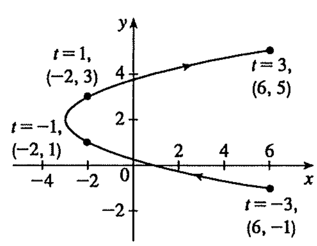
(b) $x = 3t + 2 \;\Rightarrow\; 3t = x - 2 \;\Rightarrow\; t = \frac{1}{3}x - \frac{2}{3}$, so

$y = 2t + 3 = 2\left(\frac{1}{3}x - \frac{2}{3}\right) + 3 = \frac{2}{3}x - \frac{4}{3} + 3 \;\Rightarrow\; y = \frac{2}{3}x + \frac{5}{3}$

9. $x = t^2 - 3, \quad y = t + 2, \quad -3 \le t \le 3$

(a)

t	-3	-1	1	3
x	6	-2	-2	6
y	-1	1	3	5



(b) $y = t + 2 \;\Rightarrow\; t = y - 2$, so

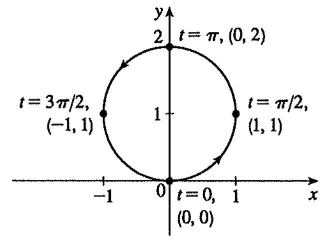
$x = t^2 - 3 = (y - 2)^2 - 3 = y^2 - 4y + 4 - 3 \;\Rightarrow$

$x = y^2 - 4y + 1, -1 \le y \le 5$

10. $x = \sin t, \quad y = 1 - \cos t, \quad 0 \le t \le 2\pi$

(a)

t	0	$\pi/2$	π	$3\pi/2$	2π
x	0	1	0	-1	0
y	0	1	2	1	0



(b) $x = \sin t, y = 1 - \cos t$ [or $y - 1 = -\cos t$] $\Rightarrow$

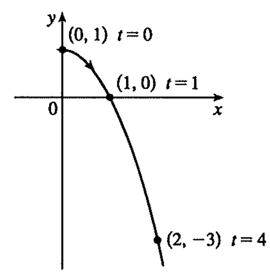
$x^2 + (y-1)^2 = (\sin t)^2 + (-\cos t)^2 \;\Rightarrow\; x^2 + (y-1)^2 = 1.$

As t varies from 0 to 2π, the circle with center $(0, 1)$ and radius 1 is traced out.

11. $x = \sqrt{t}, \; y = 1 - t$

(a)

t	0	1	2	3	4
x	0	1	1.414	1.732	2
y	1	0	-1	-2	-3



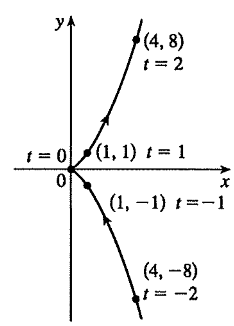
(b) $x = \sqrt{t} \;\Rightarrow\; t = x^2 \;\Rightarrow\; y = 1 - t = 1 - x^2$. Since $t \ge 0$, $x \ge 0$.

So the curve is the right half of the parabola $y = 1 - x^2$.

12. $x = t^2, \quad y = t^3$

(a)

t	-2	-1	0	1	2
x	4	1	0	1	4
y	-8	-1	0	1	8

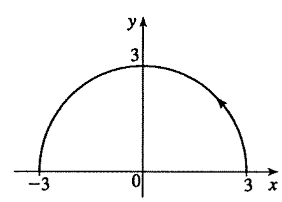


(b) $y = t^3 \;\Rightarrow\; t = \sqrt[3]{y} \;\Rightarrow\; x = t^2 = \left(\sqrt[3]{y}\right)^2 = y^{2/3}. \quad t \in \mathbb{R}, y \in \mathbb{R}, x \ge 0.$

13. (a) $x = 3\cos t, \quad y = 3\sin t, \quad 0 \le t \le \pi$

$x^2 + y^2 = 9\cos^2 t + 9\sin^2 t = 9(\cos^2 t + \sin^2 t) = 9$, which is the equation of a circle with radius 3. For $0 \le t \le \pi/2$, we have $3 \ge x \ge 0$ and $0 \le y \le 3$. For $\pi/2 < t \le \pi$, we have $0 > x \ge -3$ and $3 > y \ge 0$. Thus, the curve is the top half of the circle $x^2 + y^2 = 9$ traced counterclockwise.

(b)



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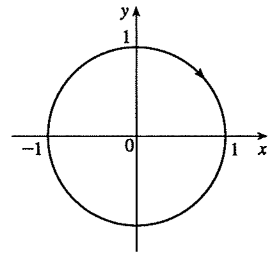
14. (a) $x = \sin 4\theta$, $\quad y = \cos 4\theta$, $\quad 0 \le \theta \le \pi/2$

$x^2 + y^2 = \sin^2 4\theta + \cos^2 4\theta = 1$, which is the equation of a circle with radius 1. When $\theta = 0$, we have $x = 0$ and $y = 1$. For $0 \le \theta \le \pi/4$, we have $x \ge 0$. For $\pi/4 < \theta \le \pi/2$, we have $x \le 0$. Thus, the curve is the circle $x^2 + y^2 = 1$ traced clockwise starting at $(0, 1)$.

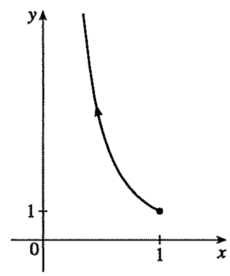
(b)



15. (a) $x = \cos\theta$, $\quad y = \sec^2\theta$, $\quad 0 \le \theta < \pi/2$.

$y = \sec^2\theta = \dfrac{1}{\cos^2\theta} = \dfrac{1}{x^2}$. For $0 \le \theta < \pi/2$, we have $1 \ge x > 0$ and $1 \le y$.

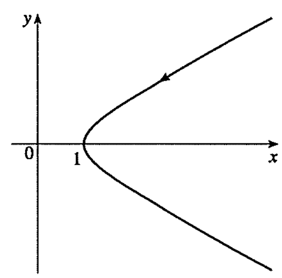
(b)



16. (a) $x = \csc t$, $y = \cot t$, $0 < t < \pi$

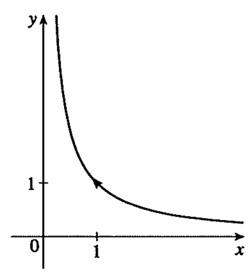
$y^2 - x^2 = \cot^2 t - \csc^2 t = 1$. For $0 < t < \pi$, we have $x > 1$. Thus, the curve is the right branch of the hyperbola $y^2 - x^2 = 1$.

(b)



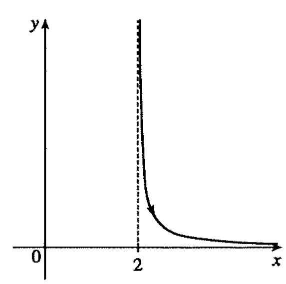
17. (a) $y = e^t = 1/e^{-t} = 1/x$ for $x > 0$ since $x = e^{-t}$. Thus, the curve is the portion of the hyperbola $y = 1/x$ with $x > 0$.

(b)



18. (a) $x = t + 2 \;\Rightarrow\; t = x - 2$. $y = 1/t = 1/(x - 2)$. For $t > 0$, we have $x > 2$ and $y > 0$. Thus, the curve is the portion of the hyperbola $y = 1/(x - 2)$ with $x > 2$.

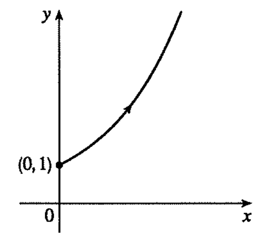
(b)



19. (a) $x = \ln t$, $y = \sqrt{t}$, $t \geq 1$.

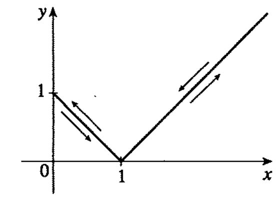
$x = \ln t \;\Rightarrow\; t = e^x \;\Rightarrow\; y = \sqrt{t} = e^{x/2}$, $x \geq 0$.

(b)



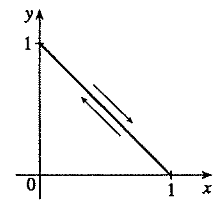
20. (a) $x = |t|$, $y = \Big|1 - |t|\Big| = |1 - x|$. For all t, we have $x \geq 0$ and $y \geq 0$. Thus, the curve is the portion of the absolute value function $y = |1 - x|$ with $x \geq 0$.

(b)



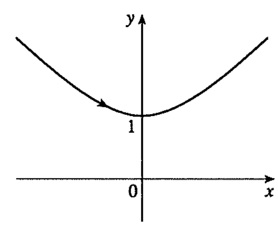
21. (a) $x = \sin^2 t$, $y = \cos^2 t$. $x + y = \sin^2 t + \cos^2 t = 1$. For all t, we have $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Thus, the curve is the portion of the line $x + y = 1$ or $y = -x + 1$ in the first quadrant.

(b)



22. (a) $x = \sinh t$, $y = \cosh t \;\Rightarrow\; y^2 - x^2 = \cosh^2 t - \sinh^2 t = 1$. Since $y = \cosh t \geq 1$, we have the upper branch of the hyperbola $y^2 - x^2 = 1$.

(b)



23. The parametric equations $x = 5\cos t$ and $y = -5\sin t$ both have period 2π. When $t = 0$, we have $x = 5$ and $y = 0$. When $t = \pi/2$, we have $x = 0$ and $y = -5$. This is one-fourth of a circle. Thus, the object completes one revolution in $4 \cdot \frac{\pi}{2} = 2\pi$ seconds following a clockwise path.

24. The parametric equations $x = 3\sin\left(\frac{\pi}{4}t\right)$ and $y = 3\cos\left(\frac{\pi}{4}t\right)$ both have period $\dfrac{2\pi}{\pi/4} = 8$. When $t = 0$, we have $x = 0$ and $y = 3$. When $t = 2$, we have $x = 3$ and $y = 0$. This is one-fourth of a circle. Thus, the object completes one revolution in $4 \cdot 2 = 8$ seconds following a clockwise path.

25. $x = 5 + 2\cos \pi t$, $y = 3 + 2\sin \pi t \;\Rightarrow\; \cos \pi t = \dfrac{x-5}{2}$, $\sin \pi t = \dfrac{y-3}{2}$. $\cos^2(\pi t) + \sin^2(\pi t) = 1 \;\Rightarrow$ $\left(\dfrac{x-5}{2}\right)^2 + \left(\dfrac{y-3}{2}\right)^2 = 1$. The motion of the particle takes place on a circle centered at $(5, 3)$ with a radius 2. As t goes from 1 to 2, the particle starts at the point $(3, 3)$ and moves counterclockwise along the circle $\left(\dfrac{x-5}{2}\right)^2 + \left(\dfrac{y-3}{2}\right)^2 = 1$ to $(7, 3)$ [one-half of a circle].

26. $x = 2 + \sin t$, $y = 1 + 3\cos t \;\Rightarrow\; \sin t = x - 2$, $\cos t = \dfrac{y-1}{3}$. $\sin^2 t + \cos^2 t = 1 \;\Rightarrow\; (x-2)^2 + \left(\dfrac{y-1}{3}\right)^2 = 1$. The motion of the particle takes place on an ellipse centered at $(2, 1)$. As t goes from $\pi/2$ to 2π, the particle starts at the point $(3, 1)$ and moves counterclockwise three-fourths of the way around the ellipse to $(2, 4)$.

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27. $x = 5\sin t$, $y = 2\cos t$ $\Rightarrow$ $\sin t = \frac{x}{5}$, $\cos t = \frac{y}{2}$. $\sin^2 t + \cos^2 t = 1$ $\Rightarrow$ $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$. The motion of the particle takes place on an ellipse centered at $(0, 0)$. As t goes from $-\pi$ to 5π, the particle starts at the point $(0, -2)$ and moves clockwise around the ellipse 3 times.

28. $y = \cos^2 t = 1 - \sin^2 t = 1 - x^2$. The motion of the particle takes place on the parabola $y = 1 - x^2$. As t goes from -2π to $-\pi$, the particle starts at the point $(0, 1)$, moves to $(1, 0)$, and goes back to $(0, 1)$. As t goes from $-\pi$ to 0, the particle moves to $(-1, 0)$ and goes back to $(0, 1)$. The particle repeats this motion as t goes from 0 to 2π.

29. We must have $1 \le x \le 4$ and $2 \le y \le 3$. So the graph of the curve must be contained in the rectangle $[1, 4]$ by $[2, 3]$.

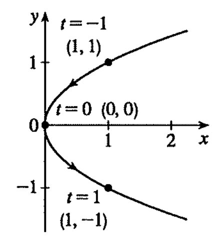
30. (a) From the first graph, we have $1 \le x \le 2$. From the second graph, we have $-1 \le y \le 1$. The only choice that satisfies either of those conditions is III.

(b) From the first graph, the values of x cycle through the values from -2 to 2 four times. From the second graph, the values of y cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

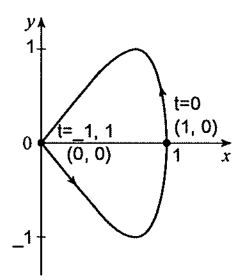
(c) From the first graph, the values of x cycle through the values from -2 to 2 three times. From the second graph, we have $0 \le y \le 2$. Choice IV satisfies these conditions.

(d) From the first graph, the values of x cycle through the values from -2 to 2 two times. From the second graph, the values of y do the same thing. Choice II satisfies these conditions.

31. When $t = -1$, $(x, y) = (1, 1)$. As t increases to 0, x and y both decrease to 0. As t increases from 0 to 1, x increases from 0 to 1 and y decreases from 0 to -1. As t increases beyond 1, x continues to increase and y continues to decrease. For $t < -1$, x and y are both positive and decreasing. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.



32. When $t = -1$, $(x, y) = (0, 0)$. As t increases to 0, x increases from 0 to 1, while y first decreases to -1 and then increases to 0. As t increases from 0 to 1, x decreases from 1 to 0, while y first increases to 1 and then decreases to 0. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.



33. When $t = -1$, $(x, y) = (0, 1)$. As t increases to 0, x increases from 0 to 1 and y decreases from 1 to 0. As t increases from 0 to 1, the curve is retraced in the opposite direction with x decreasing from 1 to 0 and y increasing from 0 to 1. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.