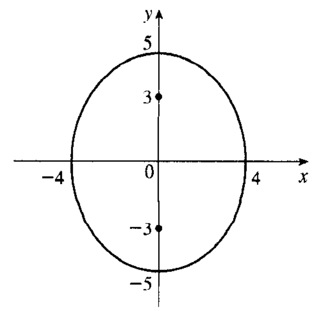
HW #8, PART III, SEC 10.5 SOLUTIONS

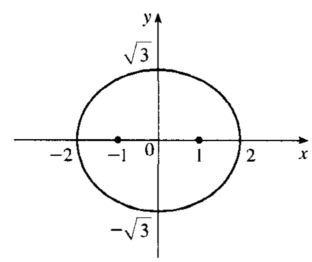
11. $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1 \quad \Rightarrow \quad a = \sqrt{25} = 5,\, b = \sqrt{16} = 4,$

$c = \sqrt{a^2 - b^2} = \sqrt{25 - 16} = \sqrt{9} = 3$. The ellipse is centered at $(0, 0)$ with vertices $(0, \pm 5)$. The foci are $(0, \pm 3)$.



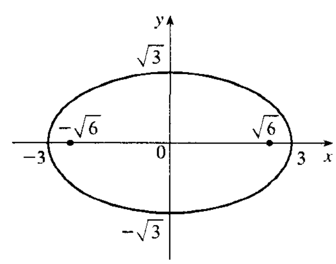
12. $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1 \quad \Rightarrow \quad a = \sqrt{4} = 2,\, b = \sqrt{3},$

$c = \sqrt{a^2 - b^2} = \sqrt{4 - 3} = \sqrt{1} = 1$. The ellipse is centered at $(0, 0)$ with vertices $(\pm 2, 0)$. The foci are $(\pm 1, 0)$.



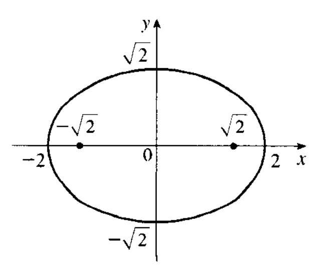
13. $x^2 + 3y^2 = 9 \quad \Leftrightarrow \quad \dfrac{x^2}{9} + \dfrac{y^2}{3} = 1 \quad \Rightarrow \quad a = \sqrt{9} = 3,\, b = \sqrt{3},$

$c = \sqrt{a^2 - b^2} = \sqrt{9 - 3} = \sqrt{6}$. The ellipse is centered at $(0, 0)$ with vertices $(\pm 3, 0)$. The foci are $(\pm\sqrt{6}, 0)$.



14. $x^2 = 4 - 2y^2 \quad \Leftrightarrow \quad x^2 + 2y^2 = 4 \quad \Leftrightarrow \quad \dfrac{x^2}{4} + \dfrac{y^2}{2} = 1 \quad \Rightarrow$

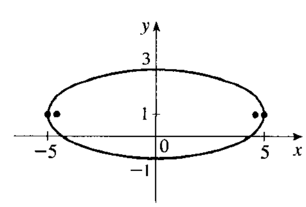
$a = \sqrt{4} = 2,\, b = \sqrt{2},\, c = \sqrt{a^2 - b^2} = \sqrt{4 - 2} = \sqrt{2}$. The ellipse is centered at $(0, 0)$ with vertices $(\pm 2, 0)$. The foci are $(\pm\sqrt{2}, 0)$.



15. $4x^2 + 25y^2 - 50y = 75 \quad \Leftrightarrow \quad 4x^2 + 25(y^2 - 2y + 1) = 75 + 25 \quad \Leftrightarrow$

$4x^2 + 25(y-1)^2 = 100 \quad \Leftrightarrow \quad \dfrac{x^2}{25} + \dfrac{(y-1)^2}{4} = 1 \quad \Rightarrow \quad a = \sqrt{25} = 5,$

$b = \sqrt{4} = 2,\, c = \sqrt{25 - 4} = \sqrt{21}$. The ellipse is centered at $(0, 1)$ with vertices $(\pm 5, 1)$. The foci are $(\pm\sqrt{21}, 1)$.

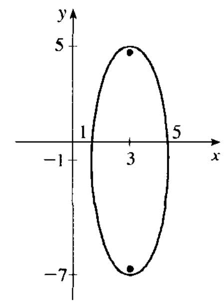


16. $9x^2 - 54x + y^2 + 2y + 46 = 0 \quad \Leftrightarrow$

$9(x^2 - 6x + 9) + y^2 + 2y + 1 = -46 + 81 + 1 \quad \Leftrightarrow$

$9(x-3)^2 + (y+1)^2 = 36 \quad \Leftrightarrow \quad \dfrac{(x-3)^2}{4} + \dfrac{(y+1)^2}{36} = 1 \quad \Rightarrow$

$a = \sqrt{36} = 6$, $b = \sqrt{4} = 2$, $c = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$. The ellipse is centered at $(3, -1)$ with vertices $(3, 5)$ and $(3, -7)$. The foci are $\left(3, -1 \pm 4\sqrt{2}\right)$.



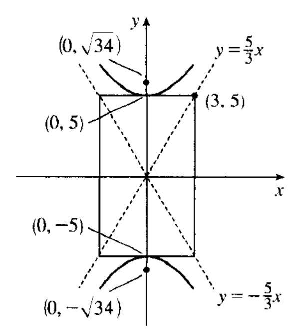
17. The center is $(0, 0)$, $a = 3$, and $b = 2$, so an equation is $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$. $c = \sqrt{a^2 - b^2} = \sqrt{5}$, so the foci are $\left(0, \pm\sqrt{5}\right)$.

18. The ellipse is centered at $(2, 1)$, with $a = 3$ and $b = 2$. An equation is $\dfrac{(x-2)^2}{9} + \dfrac{(y-1)^2}{4} = 1$. $c = \sqrt{a^2 - b^2} = \sqrt{5}$, so the foci are $\left(2 \pm \sqrt{5}, 1\right)$.

19. $\dfrac{y^2}{25} - \dfrac{x^2}{9} = 1 \quad \Rightarrow \quad a = 5, b = 3, c = \sqrt{a^2 + b^2} = \sqrt{25 + 9} = \sqrt{34} \quad \Rightarrow$

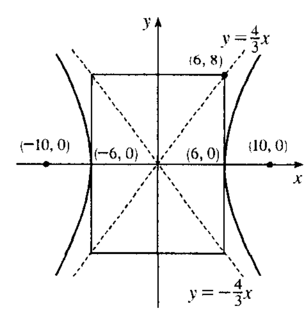
center $(0, 0)$, vertices $(0, \pm 5)$, foci $\left(0, \pm\sqrt{34}\right)$, asymptotes $y = \pm\frac{5}{3}x$.

Note: It is helpful to draw a $2a$-by-$2b$ rectangle whose center is the center of the hyperbola. The asymptotes are the extended diagonals of the rectangle.



20. $\dfrac{x^2}{36} - \dfrac{y^2}{64} = 1 \quad \Rightarrow \quad a = 6, b = 8, c = \sqrt{a^2 + b^2} = \sqrt{36 + 64} = 10 \quad \Rightarrow$

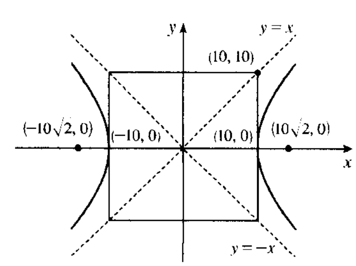
center $(0, 0)$, vertices $(\pm 6, 0)$, foci $(\pm 10, 0)$, asymptotes $y = \pm\frac{8}{6}x = \pm\frac{4}{3}x$



21. $x^2 - y^2 = 100 \quad \Leftrightarrow \quad \dfrac{x^2}{100} - \dfrac{y^2}{100} = 1 \quad \Rightarrow \quad a = b = 10$,

$c = \sqrt{100 + 100} = 10\sqrt{2} \quad \Rightarrow \quad$ center $(0, 0)$, vertices $(\pm 10, 0)$,

foci $\left(\pm 10\sqrt{2}, 0\right)$, asymptotes $y = \pm\frac{10}{10}x = \pm x$



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39. The ellipse with foci $(\pm 2, 0)$ and vertices $(\pm 5, 0)$ has center $(0, 0)$ and a horizontal major axis, with $a = 5$ and $c = 2$, so $b^2 = a^2 - c^2 = 25 - 4 = 21$. An equation is $\frac{x^2}{25} + \frac{y^2}{21} = 1$.

40. The ellipse with foci $(0, \pm\sqrt{2})$ and vertices $(0, \pm 2)$ has center $(0, 0)$ and a vertical major axis, with $a = 2$ and $c = \sqrt{2}$, so $b^2 = a^2 - c^2 = 4 - 2 = 2$. An equation is $\frac{x^2}{2} + \frac{y^2}{4} = 1$.

41. Since the vertices are $(0, 0)$ and $(0, 8)$, the ellipse has center $(0, 4)$ with a vertical axis and $a = 4$. The foci at $(0, 2)$ and $(0, 6)$ are 2 units from the center, so $c = 2$ and $b = \sqrt{a^2 - c^2} = \sqrt{4^2 - 2^2} = \sqrt{12}$. An equation is $\frac{(x-0)^2}{b^2} + \frac{(y-4)^2}{a^2} = 1 \Rightarrow \frac{x^2}{12} + \frac{(y-4)^2}{16} = 1$.

42. Since the foci are $(0, -1)$ and $(8, -1)$, the ellipse has center $(4, -1)$ with a horizontal axis and $c = 4$. The vertex $(9, -1)$ is 5 units from the center, so $a = 5$ and $b = \sqrt{a^2 - c^2} = \sqrt{5^2 - 4^2} = \sqrt{9}$. An equation is $\frac{(x-4)^2}{a^2} + \frac{(y+1)^2}{b^2} = 1 \Rightarrow \frac{(x-4)^2}{25} + \frac{(y+1)^2}{9} = 1$.

43. An equation of an ellipse with center $(-1, 4)$ and vertex $(-1, 0)$ is $\frac{(x+1)^2}{b^2} + \frac{(y-4)^2}{4^2} = 1$. The focus $(-1, 6)$ is 2 units from the center, so $c = 2$. Thus, $b^2 + 2^2 = 4^2 \Rightarrow b^2 = 12$, and the equation is $\frac{(x+1)^2}{12} + \frac{(y-4)^2}{16} = 1$.

44. Foci $F_1(-4, 0)$ and $F_2(4, 0) \Rightarrow c = 4$ and an equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The ellipse passes through $P(-4, 1.8)$, so $2a = |PF_1| + |PF_2| \Rightarrow 2a = 1.8 + \sqrt{8^2 + (1.8)^2} \Rightarrow 2a = 1.8 + 8.2 \Rightarrow a = 5$. $b^2 = a^2 - c^2 = 25 - 16 = 9$ and the equation is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

45. An equation of a hyperbola with vertices $(\pm 3, 0)$ is $\frac{x^2}{3^2} - \frac{y^2}{b^2} = 1$. Foci $(\pm 5, 0) \Rightarrow c = 5$ and $3^2 + b^2 = 5^2 \Rightarrow b^2 = 25 - 9 = 16$, so the equation is $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

46. An equation of a hyperbola with vertices $(0, \pm 2)$ is $\frac{y^2}{2^2} - \frac{x^2}{b^2} = 1$. Foci $(0, \pm 5) \Rightarrow c = 5$ and $2^2 + b^2 = 5^2 \Rightarrow b^2 = 25 - 4 = 21$, so the equation is $\frac{y^2}{4} - \frac{x^2}{21} = 1$.

47. The center of a hyperbola with vertices $(-3, -4)$ and $(-3, 6)$ is $(-3, 1)$, so $a = 5$ and an equation is $\frac{(y-1)^2}{5^2} - \frac{(x+3)^2}{b^2} = 1$. Foci $(-3, -7)$ and $(-3, 9) \Rightarrow c = 8$, so $5^2 + b^2 = 8^2 \Rightarrow b^2 = 64 - 25 = 39$ and the equation is $\frac{(y-1)^2}{25} - \frac{(x+3)^2}{39} = 1$.

48. The center of a hyperbola with vertices $(-1, 2)$ and $(7, 2)$ is $(3, 2)$, so $a = 4$ and an equation is $\frac{(x-3)^2}{4^2} - \frac{(y-2)^2}{b^2} = 1$. Foci $(-2, 2)$ and $(8, 2) \Rightarrow c = 5$, so $4^2 + b^2 = 5^2 \Rightarrow b^2 = 25 - 16 = 9$ and the equation is $\frac{(x-3)^2}{16} - \frac{(y-2)^2}{9} = 1$.