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6. Let $u = 1 + \frac{1}{x}$. Then $du = -\frac{1}{x^2}\,dx$ and $\frac{1}{x^2}\,dx = -du$, so

$$\int \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx = \int \sqrt{u}\,(-du) = -\frac{2}{3}u^{3/2} + C = -\frac{2}{3}\left(1+\frac{1}{x}\right)^{3/2} + C.$$

7. Let $u = \sqrt{t}$. Then $du = \frac{1}{2\sqrt{t}}\,dt$ and $\frac{1}{\sqrt{t}}\,dt = 2\,du$, so $\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt = \int \cos u\,(2\,du) = 2\sin u + C = 2\sin\sqrt{t} + C$.

8. Let $u = z - 1$. Then $z = u + 1$ and $du = dz$, so

$$\begin{aligned}\int z\sqrt{z-1}\,dz &= \int (u+1)\sqrt{u}\,du = \int (u^{3/2} + u^{1/2})\,du = \tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2} + C \\ &= \tfrac{2}{5}(z-1)^{5/2} + \tfrac{2}{3}(z-1)^{3/2} + C\end{aligned}$$

9. Let $u = 1 - x^2$. Then $du = -2x\,dx$ and $x\,dx = -\frac{1}{2}\,du$, so

$\int x\sqrt{1-x^2}\,dx = \int \sqrt{u}\,\left(-\frac{1}{2}\,du\right) = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{3/2} + C.$

10. Let $u = 5 - 3x$. Then $du = -3\,dx$ and $dx = -\frac{1}{3}\,du$, so

$\int (5-3x)^{10}\,dx = \int u^{10}\left(-\frac{1}{3}\,du\right) = -\frac{1}{3}\cdot\frac{1}{11}u^{11} + C = -\frac{1}{33}(5-3x)^{11} + C.$

11. Let $u = -t^4$. Then $du = -4t^3\,dt$ and $t^3\,dt = -\frac{1}{4}\,du$, so $\int t^3e^{-t^4}\,dt = \int e^u\left(-\frac{1}{4}\,du\right) = -\frac{1}{4}e^u + C = -\frac{1}{4}e^{-t^4} + C$.

12. Let $u = 1 + \cos t$. Then $du = -\sin t\,dt$ and $\sin t\,dt = -du$, so

$\int \sin t\sqrt{1+\cos t}\,dt = \int \sqrt{u}\,(-du) = -\frac{2}{3}u^{3/2} + C = -\frac{2}{3}(1+\cos t)^{3/2} + C.$

13. Let $u = \frac{\pi}{3}t$. Then $du = \frac{\pi}{3}\,dt$ and $dt = \frac{3}{\pi}\,du$, so

$$\int \sin\left(\frac{\pi t}{3}\right)dt = \int \sin u\left(\frac{3}{\pi}\,du\right) = \frac{3}{\pi}\cdot(-\cos u) + C = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + C.$$

14. Let $u = 2\theta$. Then $du = 2\,d\theta$ and $d\theta = \frac{1}{2}\,du$, so $\int \sec^2 2\theta\,d\theta = \int \sec^2 u\left(\frac{1}{2}\,du\right) = \frac{1}{2}\tan u + C = \frac{1}{2}\tan 2\theta + C$.

15. Let $u = 4x + 7$. Then $du = 4\,dx$ and $dx = \frac{1}{4}\,du$, so $\int \frac{dx}{4x+7} = \int \frac{1}{u}\left(\frac{1}{4}\,du\right) = \frac{1}{4}\ln|u| + C = \frac{1}{4}\ln|4x+7| + C$.

16. Let $u = 4 - y^3$. Then $du = -3y^2\,dy$ and $y^2\,dy = -\frac{1}{3}\,du$, so

$\int y^2(4-y^3)^{2/3}\,dy = \int u^{2/3}\left(-\frac{1}{3}\,du\right) = -\frac{1}{3}\cdot\frac{3}{5}u^{5/3} + C = -\frac{1}{5}(4-y^3)^{5/3} + C.$

17. Let $u = 1 + \sin\theta$. Then $du = \cos\theta\,d\theta$, so

$$\int \frac{\cos\theta}{1+\sin\theta}\,d\theta = \int \frac{1}{u}\,du = \ln|u| + C = \ln|1+\sin\theta| + C = \ln(1+\sin\theta) + C\ \text{[since } 1+\sin\theta \text{ is nonnegative]}.$$

18. Let $u = z^3 + 1$. Then $du = 3z^2\,dz$ and $\frac{1}{3}\,du = z^2\,dz$, so

$$\int \frac{z^2}{z^3+1}\,dz = \int \frac{1}{u}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|z^3+1| + C.$$

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19. Let $u = \cos\theta$. Then $du = -\sin\theta\,d\theta$ and $\sin\theta\,d\theta = -du$, so
$\int \cos^3\theta\,\sin\theta\,d\theta = \int u^3(-du) = -\frac{1}{4}u^4 + C = -\frac{1}{4}\cos^4\theta + C.$

20. Let $u = -5r$. Then $du = -5\,dr$ and $dr = -\frac{1}{5}\,du$, so $\int e^{-5r}\,dr = \int e^u\left(-\frac{1}{5}\,du\right) = -\frac{1}{5}e^u + C = -\frac{1}{5}e^{-5r} + C.$

21. Let $x = 1 - e^u$. Then $dx = -e^u\,du$ and $e^u\,du = -dx$, so
$$\int \frac{e^u}{(1-e^u)^2}\,du = \int \frac{1}{x^2}(-dx) = -\int x^{-2}\,dx = -(-x^{-1}) + C = \frac{1}{x} + C = \frac{1}{1-e^u} + C.$$

22. Let $u = 1/x$. Then $du = -\dfrac{1}{x^2}\,dx$ and $\dfrac{1}{x^2}\,dx = -du$, so $\displaystyle\int \frac{\sin(1/x)}{x^2}\,dx = \int \sin u\,(-du) = \cos u + C = \cos(1/x) + C.$

23. Let $u = 3ax + bx^3$. Then $du = (3a + 3bx^2)\,dx = 3(a + bx^2)\,dx$, so
$$\int \frac{a + bx^2}{\sqrt{3ax + bx^3}}\,dx = \int \frac{\frac{1}{3}\,du}{u^{1/2}} = \frac{1}{3}\int u^{-1/2}\,du = \tfrac{1}{3}\cdot 2u^{1/2} + C = \tfrac{2}{3}\sqrt{3ax + bx^3} + C.$$

24. Let $u = 3t^2 + 6t - 5$. Then $du = (6t + 6)\,dt = 6(t + 1)\,dt$ and $(t + 1)\,dt = \frac{1}{6}\,du$, so
$$\int \frac{t+1}{3t^2 + 6t - 5}\,dt = \int \frac{1}{u}\left(\frac{1}{6}\,du\right) = \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln\left|3t^2 + 6t - 5\right| + C.$$

25. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, so $\displaystyle\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}(\ln x)^3 + C.$

26. Let $u = \cos x$. Then $du = -\sin x\,dx$ and $-du = \sin x\,dx$, so
$\int \sin x\,\sin(\cos x)\,dx = \int \sin u\,(-du) = (-\cos u)(-1) + C = \cos(\cos x) + C.$

27. Let $u = \tan\theta$. Then $du = \sec^2\theta\,d\theta$, so $\int \sec^2\theta\,\tan^3\theta\,d\theta = \int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}\tan^4\theta + C.$

28. Let $u = x + 2$. Then $du = dx$ and $x = u - 2$, so
$\int x\sqrt{x+2}\,dx = \int (u-2)\sqrt{u}\,du = \int (u^{3/2} - 2u^{1/2})\,du = \frac{2}{5}u^{5/2} - 2\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C.$

29. Let $u = x^2 + \dfrac{2}{x}$. Then $du = \left(2x - \dfrac{2}{x^2}\right)dx = 2\left(x - \dfrac{1}{x^2}\right)dx$ and $\left(x - \dfrac{1}{x^2}\right)dx = \dfrac{1}{2}\,du$, so
$$\int \left(x - \frac{1}{x^2}\right)\left(x^2 + \frac{2}{x}\right)^5 dx = \int u^5\left(\frac{1}{2}\,du\right) = \frac{1}{2}\cdot\frac{1}{6}u^6 + C = \frac{1}{12}\left(x^2 + \frac{2}{x}\right)^6 + C.$$

30. Let $u = ax + b$. Then $du = a\,dx$ and $dx = (1/a)\,du$, so
$$\int \frac{dx}{ax+b} = \int \frac{(1/a)\,du}{u} = \frac{1}{a}\int \frac{1}{u}\,du = \frac{1}{a}\ln|u| + C = \frac{1}{a}\ln|ax + b| + C.$$

31. Let $u = 2 + 3e^r$. Then $du = 3e^r\,dr$ and $e^r\,dr = \frac{1}{3}\,du$, so
$\int e^r(2 + 3e^r)^{3/2}\,dr = \int u^{3/2}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\cdot\frac{2}{5}u^{5/2} + C = \frac{2}{15}(2 + 3e^r)^{5/2} + C.$

32. Let $u = \arcsin x$. Then $du = \dfrac{1}{\sqrt{1-x^2}}\,dx$, so $\displaystyle\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx = \int e^u\,du = e^u + C = e^{\arcsin x} + C.$

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44. Let $u = 1 + \tan t$. Then $du = \sec^2 t\,dt$, so

$$\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}} = \int \frac{\sec^2 t\,dt}{\sqrt{1+\tan t}} = \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{1+\tan t} + C.$$

45. $\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\,dx = 2\int \frac{\sin x\cos x}{1+\cos^2 x}\,dx = 2I$. Let $u = \cos x$. Then $du = -\sin x\,dx$, so

$$2I = -2\int \frac{u\,du}{1+u^2} = -2\cdot\tfrac{1}{2}\ln(1+u^2) + C = -\ln(1+u^2) + C = -\ln(1+\cos^2 x) + C.$$

Or: Let $u = 1 + \cos^2 x$.

46. Let $u = \cos x$. Then $du = -\sin x\,dx$ and $\sin x\,dx = -du$, so

$$\int \frac{\sin x}{1+\cos^2 x}\,dx = \int \frac{-du}{1+u^2} = -\tan^{-1} u + C = -\tan^{-1}(\cos x) + C.$$

47. $\displaystyle\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$. Let $u = \sin x$. Then $du = \cos x\,dx$, so $\displaystyle\int \cot x\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sin x| + C$.

48. Let $u = \ln t$. Then $du = \dfrac{1}{t}\,dt$, so $\displaystyle\int \frac{\cos(\ln t)}{t}\,dt = \int \cos u\,du = \sin u + C = \sin(\ln t) + C$.

49. Let $u = \sin^{-1} x$. Then $du = \dfrac{1}{\sqrt{1-x^2}}\,dx$, so $\displaystyle\int \frac{dx}{\sqrt{1-x^2}\sin^{-1} x} = \int \frac{1}{u}\,du = \ln|u| + C = \ln\left|\sin^{-1} x\right| + C$.

50. Let $u = x^2$. Then $du = 2x\,dx$, so $\displaystyle\int \frac{x}{1+x^4}\,dx = \int \frac{\frac{1}{2}\,du}{1+u^2} = \tfrac{1}{2}\tan^{-1} u + C = \tfrac{1}{2}\tan^{-1}(x^2) + C$.

51. Let $u = 1 + x^2$. Then $du = 2x\,dx$, so

$$\begin{aligned}\int \frac{1+x}{1+x^2}\,dx &= \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx = \tan^{-1} x + \int \frac{\frac{1}{2}\,du}{u} = \tan^{-1} x + \tfrac{1}{2}\ln|u| + C \\ &= \tan^{-1} x + \tfrac{1}{2}\ln\left|1+x^2\right| + C = \tan^{-1} x + \tfrac{1}{2}\ln(1+x^2) + C \quad [\text{since } 1 + x^2 > 0].\end{aligned}$$

52. Let $u = 2 + x$. Then $du = dx$, $x = u - 2$, and $x^2 = (u-2)^2$, so

$$\begin{aligned}\textstyle\int x^2\sqrt{2+x}\,dx &= \textstyle\int (u-2)^2\sqrt{u}\,du = \int (u^2 - 4u + 4)u^{1/2}\,du = \int (u^{5/2} - 4u^{3/2} + 4u^{1/2})\,du \\ &= \tfrac{2}{7}u^{7/2} - \tfrac{8}{5}u^{5/2} + \tfrac{8}{3}u^{3/2} + C = \tfrac{2}{7}(2+x)^{7/2} - \tfrac{8}{5}(2+x)^{5/2} + \tfrac{8}{3}(2+x)^{3/2} + C\end{aligned}$$

53. Let $u = 2x + 5$. Then $du = 2\,dx$ and $x = \frac{1}{2}(u-5)$, so

$$\begin{aligned}\textstyle\int x(2x+5)^8\,dx &= \textstyle\int \frac{1}{2}(u-5)u^8\left(\frac{1}{2}\,du\right) = \frac{1}{4}\int (u^9 - 5u^8)\,du \\ &= \tfrac{1}{4}\left(\tfrac{1}{10}u^{10} - \tfrac{5}{9}u^9\right) + C = \tfrac{1}{40}(2x+5)^{10} - \tfrac{5}{36}(2x+5)^9 + C\end{aligned}$$

54. Let $u = x^2 + 1$ [so $x^2 = u - 1$]. Then $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$, so

$$\begin{aligned}\textstyle\int x^3\sqrt{x^2+1}\,dx &= \textstyle\int x^2\sqrt{x^2+1}\,x\,dx = \int (u-1)\sqrt{u}\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int (u^{3/2} - u^{1/2})\,du \\ &= \tfrac{1}{2}\left(\tfrac{2}{5}u^{5/2} - \tfrac{2}{3}u^{3/2}\right) + C = \tfrac{1}{5}(x^2+1)^{5/2} - \tfrac{1}{3}(x^2+1)^{3/2} + C\end{aligned}$$

[continued]

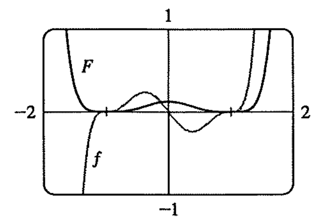
Or: Let $u = \sqrt{x^2+1}$. Then $u^2 = x^2 + 1 \Rightarrow 2u\,du = 2x\,dx \Rightarrow u\,du = x\,dx$, so

$$\int x^3\sqrt{x^2+1}\,dx = \int x^2\sqrt{x^2+1}\,x\,dx = \int (u^2-1)\,u\cdot u\,du = \int (u^4-u^2)\,du$$
$$= \tfrac{1}{5}u^5 - \tfrac{1}{3}u^3 + C = \tfrac{1}{5}(x^2+1)^{5/2} - \tfrac{1}{3}(x^2+1)^{3/2} + C$$

Note: This answer can be written as $\frac{1}{15}\sqrt{x^2+1}(3x^4+x^2-2)+C$.

55. $f(x) = x(x^2-1)^3$. $u = x^2 - 1 \Rightarrow du = 2x\,dx$, so

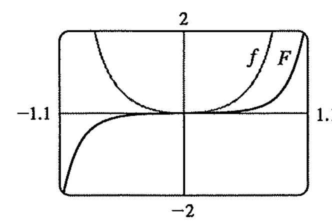
$$\int x(x^2-1)^3\,dx = \int u^3\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{8}u^4 + C = \tfrac{1}{8}(x^2-1)^4 + C$$



Where f is positive (negative), F is increasing (decreasing). Where f changes from negative to positive (positive to negative), F has a local minimum (maximum).

56. $f(\theta) = \tan^2\theta\,\sec^2\theta$. $u = \tan\theta \Rightarrow du = \sec^2\theta\,d\theta$, so

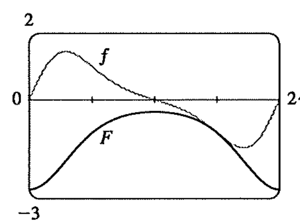
$$\int \tan^2\theta\,\sec^2\theta\,d\theta = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}\tan^3\theta + C$$



Note that f is positive and F is increasing. At $x = 0$, $f = 0$ and F has a horizontal tangent.

57. $f(x) = e^{\cos x}\sin x$. $u = \cos x \Rightarrow du = -\sin x\,dx$, so

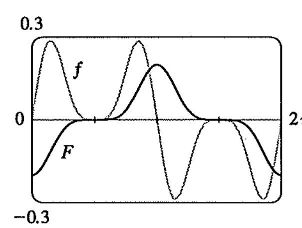
$$\int e^{\cos x}\sin x\,dx = \int e^u\,(-du) = -e^u + C = -e^{\cos x} + C$$



Note that at $x = \pi$, f changes from positive to negative and F has a local maximum. Also, both f and F are periodic with period 2π, so at $x = 0$ and at $x = 2\pi$, f changes from negative to positive and F has a local minimum.

58. $f(x) = \sin x\cos^4 x$. $u = \cos x \Rightarrow du = -\sin x\,dx$, so

$$\int \sin x\cos^4 x\,dx = \int u^4\,(-du) = -\tfrac{1}{5}u^5 + C = -\tfrac{1}{5}\cos^5 x + C$$



Note that at $x = \pi$, f changes from positive to negative and F has a local maximum. Also, both f and F are periodic with period 2π, so at $x = 0$ and at $x = 2\pi$, f changes from negative to positive and F has a local minimum.

59. Let $u = \frac{\pi}{2}t$, so $du = \frac{\pi}{2}\,dt$. When $t = 0$, $u = 0$; when $t = 1$, $u = \frac{\pi}{2}$. Thus,

$$\int_0^1 \cos(\pi t/2)\,dt = \int_0^{\pi/2}\cos u\,\left(\tfrac{2}{\pi}\,du\right) = \tfrac{2}{\pi}\left[\sin u\right]_0^{\pi/2} = \tfrac{2}{\pi}\left(\sin\tfrac{\pi}{2} - \sin 0\right) = \tfrac{2}{\pi}(1-0) = \tfrac{2}{\pi}$$

60. Let $u = 3t - 1$, so $du = 3\,dt$. When $t = 0$, $u = -1$; when $t = 1$, $u = 2$. Thus,

$$\int_0^1 (3t-1)^{50}\,dt = \int_{-1}^2 u^{50}\left(\tfrac{1}{3}\,du\right) = \tfrac{1}{3}\left[\tfrac{1}{51}u^{51}\right]_{-1}^2 = \tfrac{1}{153}\left[2^{51} - (-1)^{51}\right] = \tfrac{1}{153}(2^{51}+1)$$

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61. Let $u = 1 + 7x$, so $du = 7\,dx$. When $x = 0$, $u = 1$; when $x = 1$, $u = 8$. Thus,

$$\int_0^1 \sqrt[3]{1+7x}\,dx = \int_1^8 u^{1/3}\left(\tfrac{1}{7}\,du\right) = \tfrac{1}{7}\left[\tfrac{3}{4}u^{4/3}\right]_1^8 = \tfrac{3}{28}\left(8^{4/3} - 1^{4/3}\right) = \tfrac{3}{28}(16-1) = \tfrac{45}{28}$$

62. Let $u = \frac{1}{2}t$, so $du = \frac{1}{2}\,dt$. When $t = \frac{\pi}{3}$, $u = \frac{\pi}{6}$; when $t = \frac{2\pi}{3}$, $u = \frac{\pi}{3}$. Thus,

$$\int_{\pi/3}^{2\pi/3} \csc^2\left(\tfrac{1}{2}t\right) dt = \int_{\pi/6}^{\pi/3} \csc^2 u\,(2\,du) = 2\left[-\cot u\right]_{\pi/6}^{\pi/3} = -2\left(\cot\frac{\pi}{3} - \cot\frac{\pi}{6}\right)$$

$$= -2\left(\frac{1}{\sqrt{3}} - \sqrt{3}\right) = -2\left(\tfrac{1}{3}\sqrt{3} - \sqrt{3}\right) = \tfrac{4}{3}\sqrt{3}$$

63. Let $u = \cos t$, so $du = -\sin t\,dt$. When $t = 0$, $u = 1$; when $t = \frac{\pi}{6}$, $u = \sqrt{3}/2$. Thus,

$$\int_0^{\pi/6} \frac{\sin t}{\cos^2 t}\,dt = \int_1^{\sqrt{3}/2} \frac{1}{u^2}\,(-du) = \left[\frac{1}{u}\right]_1^{\sqrt{3}/2} = \frac{2}{\sqrt{3}} - 1.$$

64. Let $u = 2 + \sqrt{x}$. Then $du = \dfrac{1}{2\sqrt{x}}\,dx$ and $\dfrac{1}{\sqrt{x}}\,dx = 2\,du$. When $x = 1$, $u = 3$; when $x = 4$, $u = 4$. Thus,

$$\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}}\,dx = \int_3^4 \sqrt{u}\,(2\,du) = \left[\frac{4}{3}u^{3/2}\right]_3^4 = \frac{4}{3}\left(4^{3/2} - 3^{3/2}\right) = \frac{4}{3}\left(8 - 3\sqrt{3}\right) \text{ or } \frac{32}{3} - 4\sqrt{3}.$$

65. Let $u = 1/x$, so $du = -1/x^2\,dx$. When $x = 1$, $u = 1$; when $x = 2$, $u = \frac{1}{2}$. Thus,

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx = \int_1^{1/2} e^u\,(-du) = -\left[e^u\right]_1^{1/2} = -\left(e^{1/2} - e\right) = e - \sqrt{e}.$$

66. Let $u = e^x$. Then $du = e^x\,dx$. When $x = 0$, $u = 1$; when $x = 1$, $u = e$. Thus,

$$\int_0^1 \frac{e^x}{1+e^{2x}}\,dx = \int_1^e \frac{1}{1+u^2}\,du = \left[\arctan u\right]_1^e = \arctan e - \arctan 1 = \arctan e - \frac{\pi}{4}.$$

67. $\int_{-\pi/4}^{\pi/4} (x^3 + x^4 \tan x)\,dx = 0$ by Theorem 7(b), since $f(x) = x^3 + x^4 \tan x$ is an odd function.

68. Let $u = \sin x$, so $du = \cos x\,dx$. When $x = 0$, $u = 0$; when $x = \frac{\pi}{2}$, $u = 1$. Thus,

$\int_0^{\pi/2} \cos x\,\sin(\sin x)\,dx = \int_0^1 \sin u\,du = \left[-\cos u\right]_0^1 = -(\cos 1 - 1) = 1 - \cos 1.$

69. Let $u = 1 + 2x$, so $du = 2\,dx$. When $x = 0$, $u = 1$; when $x = 13$, $u = 27$. Thus,

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_1^{27} u^{-2/3}\left(\tfrac{1}{2}\,du\right) = \left[\tfrac{1}{2}\cdot 3u^{1/3}\right]_1^{27} = \tfrac{3}{2}(3-1) = 3.$$

70. Assume $a > 0$. Let $u = a^2 - x^2$, so $du = -2x\,dx$. When $x = 0$, $u = a^2$; when $x = a$, $u = 0$. Thus,

$\int_0^a x\sqrt{a^2 - x^2}\,dx = \int_{a^2}^0 u^{1/2}\left(-\tfrac{1}{2}\,du\right) = \tfrac{1}{2}\int_0^{a^2} u^{1/2}\,du = \tfrac{1}{2}\cdot\left[\tfrac{2}{3}u^{3/2}\right]_0^{a^2} = \tfrac{1}{3}a^3.$

71. Let $u = x^2 + a^2$, so $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$. When $x = 0$, $u = a^2$; when $x = a$, $u = 2a^2$. Thus,

$$\int_0^a x\sqrt{x^2 + a^2}\,dx = \int_{a^2}^{2a^2} u^{1/2}\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{2}\left[\tfrac{2}{3}u^{3/2}\right]_{a^2}^{2a^2} = \left[\tfrac{1}{3}u^{3/2}\right]_{a^2}^{2a^2} = \tfrac{1}{3}\left[(2a^2)^{3/2} - (a^2)^{3/2}\right] = \tfrac{1}{3}\left(2\sqrt{2} - 1\right)a^3$$