

HW #11, Sec 14.4 Solutions

so we use $h = \pm 10$:

$$f_v(-15, 50) \approx \frac{f(-15, 60) - f(-15, 50)}{10} = \frac{-30 - (-29)}{10} = -0.1$$

$$f_v(-15, 50) \approx \frac{f(-15, 40) - f(-15, 50)}{-10} = \frac{-27 - (-29)}{-10} = -0.2$$

Averaging these values gives $f_v(-15, 50) \approx -0.15$. The linear approximation to the wind-chill index function, then, is $f(T, v) \approx f(-15, 50) + f_T(-15, 50)(T - (-15)) + f_v(-15, 50)(v - 50) \approx -29 + (1.3)(T + 15) - (0.15)(v - 50)$. Thus when $T = -17^\circ\text{C}$ and $v = 55 \text{ km/h}$, $f(-17, 55) \approx -29 + (1.3)(-17 + 15) - (0.15)(55 - 50) = -32.35$, so we estimate the wind-chill index to be approximately -32.35°C .

$$(31) \quad m = p^5 q^3 \Rightarrow dm = \frac{\partial m}{\partial p} dp + \frac{\partial m}{\partial q} dq = 5p^4 q^3 dp + 3p^5 q^2 dq$$

$$32. \quad z = x \ln(y^2 + 1) \Rightarrow dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = \ln(y^2 + 1) dx + \frac{x}{y^2 + 1} (2y) dy = \ln(y^2 + 1) dx + \frac{2xy}{y^2 + 1} dy$$

$$(33) \quad z = e^{-2x} \cos 2\pi t \Rightarrow$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial t} dt = e^{-2x}(-2) \cos 2\pi t dx + e^{-2x}(-\sin 2\pi t)(2\pi) dt = -2e^{-2x} \cos 2\pi t dx - 2\pi e^{-2x} \sin 2\pi t dt$$

$$34. \quad u = \sqrt{x^2 + 3y^2} = (x^2 + 3y^2)^{1/2} \Rightarrow$$

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy = \frac{1}{2}(x^2 + 3y^2)^{-1/2}(2x) dx + \frac{1}{2}(x^2 + 3y^2)^{-1/2}(6y) dy = \frac{x}{\sqrt{x^2 + 3y^2}} dx + \frac{3y}{\sqrt{x^2 + 3y^2}} dy$$

$$35. \quad H = x^2 y^4 + y^3 z^5 \Rightarrow dH = \frac{\partial H}{\partial x} dx + \frac{\partial H}{\partial y} dy + \frac{\partial H}{\partial z} dz = 2xy^4 dx + (4x^2 y^3 + 3y^2 z^5) dy + 5y^3 z^4 dz$$

$$(36) \quad w = xze^{-y^2 - z^2} \Rightarrow$$

$$\begin{aligned} dw &= \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial z} dz = ze^{-y^2 - z^2} dx + xze^{-y^2 - z^2}(-2y) dy + x[z \cdot e^{-y^2 - z^2}(-2z) + e^{-y^2 - z^2} \cdot 1] dz \\ &= ze^{-y^2 - z^2} dx - 2xyze^{-y^2 - z^2} dy + x(1 - 2z^2)e^{-y^2 - z^2} dz \end{aligned}$$

$$37. \quad R = \alpha\beta^2 \cos \gamma \Rightarrow dR = \frac{\partial R}{\partial \alpha} d\alpha + \frac{\partial R}{\partial \beta} d\beta + \frac{\partial R}{\partial \gamma} d\gamma = \beta^2 \cos \gamma d\alpha + 2\alpha\beta \cos \gamma d\beta - \alpha\beta^2 \sin \gamma d\gamma$$

$$38. \quad T = \frac{v}{1 + uvw} \Rightarrow$$

$$\begin{aligned} dT &= \frac{\partial T}{\partial u} du + \frac{\partial T}{\partial v} dv + \frac{\partial T}{\partial w} dw \\ &= v(-1)(1 + uvw)^{-2}(vw) du + \frac{1(1 + uvw) - v(uw)}{(1 + uvw)^2} dv + v(-1)(1 + uvw)^{-2}(uv) dw \\ &= -\frac{v^2 w}{(1 + uvw)^2} du + \frac{1}{(1 + uvw)^2} dv - \frac{uv^2}{(1 + uvw)^2} dw \end{aligned}$$

39. $dx = \Delta x = 0.05$, $dy = \Delta y = 0.1$, $z = 5x^2 + y^2$, $z_x = 10x$, $z_y = 2y$. Thus when $x = 1$ and $y = 2$,

$$dz = z_x(1, 2) dx + z_y(1, 2) dy = (10)(0.05) + (4)(0.1) = 0.9 \text{ while}$$

$$\Delta z = f(1.05, 2.1) - f(1, 2) = 5(1.05)^2 + (2.1)^2 - 5 - 4 = 0.9225.$$

40. $dx = \Delta x = -0.04$, $dy = \Delta y = 0.05$, $z = x^2 - xy + 3y^2$, $z_x = 2x - y$, $z_y = 6y - x$. Thus when $x = 3$ and $y = -1$,

$$dz = (7)(-0.04) + (-9)(0.05) = -0.73 \text{ while } \Delta z = (2.96)^2 - (2.96)(-0.95) + 3(-0.95)^2 - (9 + 3 + 3) = -0.7189.$$

41. $dA = \frac{\partial A}{\partial x} dx + \frac{\partial A}{\partial y} dy = y dx + x dy$ and $|\Delta x| \leq 0.1$, $|\Delta y| \leq 0.1$. We use $dx = 0.1$, $dy = 0.1$ with $x = 30$, $y = 24$; then

$$\text{the maximum error in the area is about } dA = 24(0.1) + 30(0.1) = 5.4 \text{ cm}^2.$$

42. Let V be the volume. Then $V = \pi r^2 h$ and $\Delta V \approx dV = 2\pi r h dr + \pi r^2 dh$ is an estimate of the amount of metal. With

$$dr = 0.05 \text{ and } dh = 0.2 \text{ (0.1 on top, 0.1 on bottom), we get } dV = 2\pi(2)(10)(0.05) + \pi(2)^2(0.2) = 2.80\pi \approx 8.8 \text{ cm}^3.$$

43. The volume of a can is $V = \pi r^2 h$ and $\Delta V \approx dV$ is an estimate of the amount of tin. Here $dV = 2\pi r h dr + \pi r^2 dh$, so put

$$dr = 0.04, dh = 0.08 \text{ (0.04 on top, 0.04 on bottom) and then } \Delta V \approx dV = 2\pi(48)(0.04) + \pi(16)(0.08) \approx 16.08 \text{ cm}^3.$$

Thus the amount of tin is about 16 cm^3 .

44. (a) Let A be the area. Then $A = \frac{1}{2}bh$ and $\Delta A \approx dA = \frac{\partial A}{\partial b} db + \frac{\partial A}{\partial h} dh = \frac{1}{2}h db + \frac{1}{2}b dh$. We have $|\Delta h| = |\Delta b| \leq \varepsilon$.

So we take $dh = db = \varepsilon$ with $b = 28$ inches, $h = 16$ inches. The maximum error in the area is

$$dA = \frac{1}{2}(16)\varepsilon + \frac{1}{2}(28)\varepsilon = 22\varepsilon \text{ square inches.}$$

(b) With $\varepsilon = \frac{1}{4}$, we have the estimated maximum error in the area of the triangle as $22(\frac{1}{4}) = \frac{11}{2}$ square inches.

45. (a) Let V be the volume. Then $V = \pi r^2 h$ and $\Delta V \approx dV = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial h} dh = 2\pi r h dr + \pi r^2 dh$. We have

$|\Delta r| = |\Delta h| = \varepsilon$. So we take $dr = dh = \varepsilon$ with $r = 2.5$ feet and $h = 12$ feet. The maximum error in the volume is

$$dV = 2\pi(2.5)(12)\varepsilon + \pi(2.5^2)\varepsilon = 66.25\pi\varepsilon \text{ ft}^3.$$

(b) We need $66.25\pi\varepsilon \leq 1 \Rightarrow \varepsilon \lesssim 0.0048$ feet or 0.058 inches.

46. $W = 13.12 + 0.6215T - 11.37v^{0.16} + 0.3965Tv^{0.16}$, so the differential of W is

$$\begin{aligned} dW &= \frac{\partial W}{\partial T} dT + \frac{\partial W}{\partial v} dv = (0.6215 + 0.3965v^{0.16}) dT + [-11.37(0.16)v^{-0.84} + 0.3965T(0.16)v^{-0.84}] dv \\ &= (0.6215 + 0.3965v^{0.16}) dT + (-1.8192 + 0.06344T)v^{-0.84} dv \end{aligned}$$

Here we have $|\Delta T| \leq 1$, $|\Delta v| \leq 2$, so we take $dT = 1$, $dv = 2$ with $T = -11$, $v = 26$. The maximum error in the calculated

value of W is about $dW = (0.6215 + 0.3965(26)^{0.16})(1) + (-1.8192 + 0.06344(-11))(26)^{-0.84}(2) \approx 0.96$.