

HW #13, Sec. 15.2 SOLUTIONS

To find g_{yx} , we first use Fubini's Theorem to find that $\int_a^x \int_c^y f(s, t) dt ds = \int_c^y \int_a^x f(s, t) dt ds$, and then use the Fundamental Theorem twice, as above, to get $g_{yx} = f(x, y)$. So $g_{xy} = g_{yx} = f(x, y)$.

15.2 Double Integrals over General Regions

$$\begin{aligned} 1. \int_1^5 \int_0^x (8x - 2y) dy dx &= \int_1^5 [8xy - y^2]_{y=0}^{y=x} dx = \int_1^5 [8x(x) - (x)^2 - 8x(0) + (0)^2] dx \\ &= \int_1^5 7x^2 dx = \left[\frac{7}{3} x^3 \right]_1^5 = \frac{7}{3} (125 - 1) = \frac{868}{3} \end{aligned}$$

$$\begin{aligned} 2. \int_0^2 \int_0^{y^2} x^2 y dx dy &= \int_0^2 \left[\frac{1}{3} x^3 y \right]_{x=0}^{x=y^2} dy = \int_0^2 \frac{1}{3} y [(y^2)^3 - (0)^3] dy \\ &= \int_0^2 \frac{1}{3} y^7 dy = \left[\frac{1}{24} y^8 \right]_0^2 = \frac{1}{3} (32 - 0) = \frac{32}{3} \end{aligned}$$

$$\begin{aligned} 3. \int_0^1 \int_0^y x e^{y^3} dx dy &= \int_0^1 \left[\frac{1}{2} x^2 e^{y^3} \right]_{x=0}^{x=y} dy = \int_0^1 \frac{1}{2} e^{y^3} [(y)^2 - (0)^2] dy \\ &= \frac{1}{2} \int_0^1 y^2 e^{y^3} dy = \frac{1}{2} \left[\frac{1}{3} e^{y^3} \right]_0^1 = \frac{1}{2} \cdot \frac{1}{3} (e^1 - e^0) = \frac{1}{6} (e - 1) \end{aligned}$$

$$\begin{aligned} 4. \int_0^{\pi/2} \int_0^x x \sin y dy dx &= \int_0^{\pi/2} [x(-\cos y)]_{y=0}^{y=x} dx = \int_0^{\pi/2} (-x \cos x + x) dx = \int_0^{\pi/2} (x - x \cos x) dx \\ &= \left[\frac{1}{2} x^2 - (x \sin x + \cos x) \right]_0^{\pi/2} \quad (\text{by integrating by parts in the second term}) \\ &= \left(\frac{1}{2} \cdot \frac{\pi^2}{4} - \frac{\pi}{2} - 0 \right) - (0 - 0 - 1) = \frac{\pi^2}{8} - \frac{\pi}{2} + 1 \end{aligned}$$

$$5. \int_0^1 \int_0^{s^2} \cos(s^3) dt ds = \int_0^1 [t \cos(s^3)]_{t=0}^{t=s^2} ds = \int_0^1 s^2 \cos(s^3) ds = \left[\frac{1}{3} \sin(s^3) \right]_0^1 = \frac{1}{3} (\sin 1 - \sin 0) = \frac{1}{3} \sin 1$$

$$\begin{aligned} 6. \int_0^1 \int_0^{e^v} \sqrt{1+e^v} dw dv &= \int_0^1 [w \sqrt{1+e^v}]_{w=0}^{w=e^v} dv = \int_0^1 e^v \sqrt{1+e^v} dv = \left[\frac{2}{3} (1+e^v)^{3/2} \right]_0^1 \\ &= \frac{2}{3} (1+e)^{3/2} - \frac{2}{3} (1+1)^{3/2} = \frac{2}{3} (1+e)^{3/2} - \frac{4}{3} \sqrt{2} \end{aligned}$$

$$7. (a) \text{ We express the iterated integral as a Type I: } \int_0^2 \int_x^{3x-x^2} 2y dy dx. \text{ A Type II would require the sum of two integrals.}$$

$$\begin{aligned} (b) \int_0^2 \int_x^{3x-x^2} 2y dy dx &= \int_0^2 \left[y^2 \right]_{y=x}^{y=3x-x^2} dx = \int_0^2 [(3x-x^2)^2 - (x)^2] dx = \int_0^2 (8x^2 - 6x^3 + x^4) dx \\ &= \left[\frac{8}{3} x^3 - \frac{3}{2} x^4 + \frac{1}{5} x^5 \right]_0^2 = \frac{64}{3} - 24 + \frac{32}{5} = \frac{56}{15} \end{aligned}$$

$$8. (a) \text{ We express the iterated integral as a Type II: } \int_0^1 \int_y^{2-y} (x+y) dx dy. \text{ A Type I would require the sum of two integrals.}$$

$$\begin{aligned} (b) \int_0^1 \int_y^{2-y} (x+y) dx dy &= \int_0^1 \left[\frac{x^2}{2} + xy \right]_{x=y}^{x=2-y} dy = \int_0^1 \left[\left(\frac{(2-y)^2}{2} + (2-y)y \right) - \left(\frac{y^2}{2} + y^2 \right) \right] dy \\ &= \int_0^1 (2 - 2y^2) dy = \left[2y - \frac{2}{3} y^3 \right]_0^1 = 2 - \frac{2}{3} = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} 9. (a) \text{ We express the iterated integral as a Type II. A Type I would require the sum of two integrals. The curves intersect when } \sqrt{x} &= x - 2 \Rightarrow x = x^2 - 4x + 4 \Leftrightarrow 0 = x^2 - 5x + 4 \Leftrightarrow (x-4)(x-1) = 0 \Leftrightarrow x = 1 \text{ or } x = 4. \text{ The} \\ \text{point for } x = 1 \text{ is not in } D. \text{ Thus, the point of intersection of the curves is } (4, 2) \text{ and the integral is } &\int_0^2 \int_{y^2+2}^{y^2+2} xy dx dy. \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \int_0^2 \int_{y^2}^{y+2} xy \, dx \, dy &= \int_0^2 y \left[\frac{x^2}{2} \right]_{x=y^2}^{x=y+2} dy = \frac{1}{2} \int_0^2 y[(y+2)^2 - (y^2)^2] dy = \frac{1}{2} \int_0^2 [y^3 + 4y^2 + 4y - y^5] dy \\ &= \frac{1}{2} \left[\frac{1}{4}y^4 + \frac{4}{3}y^3 + 2y^2 - \frac{1}{6}y^6 \right]_0^2 = \frac{1}{2} \left(4 + \frac{32}{3} + 8 - \frac{32}{3} \right) = 6 \end{aligned}$$

10. (a) We express the iterated integral as a Type I. A Type II would require the sum of two integrals. The curves intersect when $6 - x = x^2 \Leftrightarrow x^2 + x - 6 = 0 \Leftrightarrow (x+3)(x-2) = 0 \Leftrightarrow x = -3$ or $x = 2$. The point for $x = -3$ is not in D . Thus, the point of intersection of the two curves is $(2, 4)$ and the integral is $\int_0^2 \int_{x^2}^{6-x} x \, dy \, dx$.

$$\begin{aligned} \text{(b)} \quad \int_0^2 \int_{x^2}^{6-x} x \, dy \, dx &= \int_0^2 x \left[y \right]_{y=x^2}^{y=6-x} dx = \int_0^2 x[(6-x) - x^2] dx = \int_0^2 [6x - x^2 - x^3] dx \\ &= \left[3x^2 - \frac{x^3}{3} - \frac{x^4}{4} \right]_0^2 = 12 - \frac{8}{3} - 4 = \frac{16}{3} \end{aligned}$$

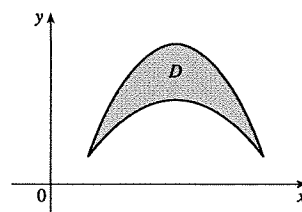
$$\begin{aligned} \textcircled{11} \quad \iint_D \frac{y}{x^2+1} \, dA &= \int_0^4 \int_0^{\sqrt{x}} \frac{y}{x^2+1} \, dy \, dx = \int_0^4 \left[\frac{1}{x^2+1} \cdot \frac{y^2}{2} \right]_{y=0}^{y=\sqrt{x}} dx = \frac{1}{2} \int_0^4 \frac{x}{x^2+1} dx \\ &= \frac{1}{2} \left[\frac{1}{2} \ln|x^2+1| \right]_0^4 = \frac{1}{4} [\ln(x^2+1)]_0^4 = \frac{1}{4} (\ln 17 - \ln 1) = \frac{1}{4} \ln 17 \end{aligned}$$

$$\begin{aligned} \textcircled{12} \quad \iint_D (2x+y) \, dA &= \int_1^2 \int_{y-1}^1 (2x+y) \, dx \, dy = \int_1^2 [x^2 + xy]_{x=y-1}^{x=1} dy = \int_1^2 [1 + y - (y-1)^2 - y(y-1)] dy \\ &= \int_1^2 (-2y^2 + 4y) dy = \left[-\frac{2}{3}y^3 + 2y^2 \right]_1^2 = \left(-\frac{16}{3} + 8 \right) - \left(-\frac{2}{3} + 2 \right) = \frac{4}{3} \end{aligned}$$

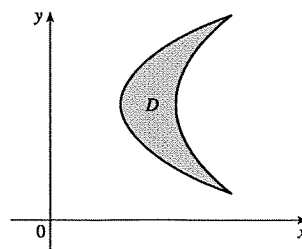
$$\begin{aligned} \textcircled{13} \quad \iint_D e^{-y^2} \, dA &= \int_0^3 \int_0^y e^{-y^2} \, dx \, dy = \int_0^3 [xe^{-y^2}]_{x=0}^{x=y} dy = \int_0^3 (ye^{-y^2} - 0) dy = \int_0^3 ye^{-y^2} dy \\ &= \left[-\frac{1}{2}e^{-y^2} \right]_0^3 = -\frac{1}{2}(e^{-9} - e^0) = \frac{1}{2}(1 - e^{-9}) \end{aligned}$$

$$\begin{aligned} 14. \quad \iint_D y\sqrt{x^2-y^2} \, dA &= \int_0^2 \int_0^x y\sqrt{x^2-y^2} \, dy \, dx = \int_0^2 \left[-\frac{1}{3}(x^2-y^2)^{3/2} \right]_{y=0}^{y=x} dx = \int_0^2 \left[0 + \frac{1}{3}(x^2)^{3/2} \right] dx \\ &= \int_0^2 \frac{1}{3}x^3 \, dx = \frac{1}{3} \cdot \frac{1}{4}x^4 \Big|_0^2 = \frac{1}{12}(16-0) = \frac{4}{3} \end{aligned}$$

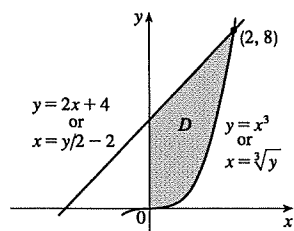
15. (a) At the right we sketch an example of a region D that can be described as lying between the graphs of two continuous functions of x (a type I region) but not as lying between graphs of two continuous functions of y (a type II region). The regions shown in Figures 6 and 8 in the text are additional examples.



- (b) Now we sketch an example of a region D that can be described as lying between the graphs of two continuous functions of y but not as lying between graphs of two continuous functions of x . The first region shown in Figure 7 is another example.



22.



By inspection, the curves $y = 2x + 4$ and $y = x^3$ intersect when $x^3 = 2x + 4 \Leftrightarrow x = 2$, so the point of intersection is $(2, 8)$. If we describe D as a type I region,

$D = \{(x, y) \mid 0 \leq x \leq 2, x^3 \leq y \leq 2x + 4\}$ and the integral is

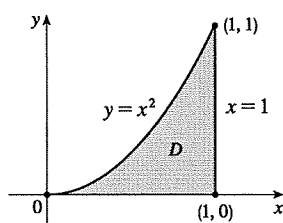
$$\iint_D 6x^2 \, dA = \int_0^2 \int_{x^3}^{2x+4} 6x^2 \, dy \, dx.$$

If we describe D as a type II region, the right boundary curve is $x = \sqrt[3]{y}$, but the left boundary curve consists of two parts, $x = 0$ for $0 \leq y \leq 4$ and $x = y/2 - 2$ for $4 \leq y \leq 8$.

In either case, the resulting iterated integrals are not difficult to evaluate, but the region D is more simply described as a type I region, giving one iterated integral rather than a sum of two, so we evaluate that integral:

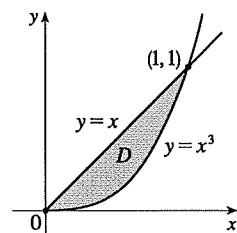
$$\begin{aligned} \int_0^2 \int_{x^3}^{2x+4} 6x^2 \, dy \, dx &= \int_0^2 [6x^2 y]_{y=x^3}^{y=2x+4} \, dx = \int_0^2 [6x^2(2x+4-x^3)] \, dx = \int_0^2 (12x^3 + 24x^2 - 6x^5) \, dx \\ &= [3x^4 + 8x^3 - x^6]_0^2 = 48 + 64 - 64 = 48 \end{aligned}$$

23.



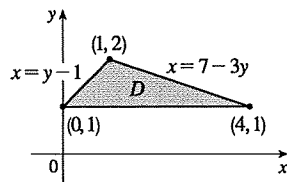
$$\begin{aligned} \int_0^1 \int_0^{x^2} x \cos y \, dy \, dx &= \int_0^1 [x \sin y]_{y=0}^{y=x^2} \, dx = \int_0^1 x \sin x^2 \, dx \\ &= -\frac{1}{2} \cos x^2 \Big|_0^1 = -\frac{1}{2} (\cos 1 - \cos 0) = \frac{1}{2} (1 - \cos 1) \end{aligned}$$

24.



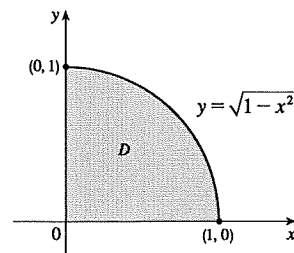
$$\begin{aligned} \iint_D (x^2 + 2y) \, dA &= \int_0^1 \int_{x^3}^x (x^2 + 2y) \, dy \, dx = \int_0^1 [x^2 y + y^2]_{y=x^3}^{y=x} \, dx \\ &= \int_0^1 (x^3 + x^2 - x^5 - x^6) \, dx = \left[\frac{1}{4} x^4 + \frac{1}{3} x^3 - \frac{1}{6} x^6 - \frac{1}{7} x^7 \right]_0^1 \\ &= \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{7} = \frac{23}{84} \end{aligned}$$

25.



$$\begin{aligned} \iint_D y^2 \, dA &= \int_1^2 \int_{y-1}^{7-3y} y^2 \, dx \, dy = \int_1^2 [xy^2]_{x=y-1}^{x=7-3y} \, dy \\ &= \int_1^2 [(7-3y) - (y-1)] y^2 \, dy = \int_1^2 (8y^2 - 4y^3) \, dy \\ &= \left[\frac{8}{3} y^3 - y^4 \right]_1^2 = \frac{64}{3} - 16 - \frac{8}{3} + 1 = \frac{11}{3} \end{aligned}$$

26.



$$\begin{aligned} \iint_D xy \, dA &= \int_0^1 \int_0^{\sqrt{1-x^2}} xy \, dy \, dx \\ &= \int_0^1 \left[\frac{1}{2} xy^2 \right]_{y=0}^{y=\sqrt{1-x^2}} \, dx = \int_0^1 \frac{1}{2} x(1-x^2) \, dx \\ &= \frac{1}{2} \int_0^1 (x - x^3) \, dx = \frac{1}{2} \left[\frac{1}{2} x^2 - \frac{1}{4} x^4 \right]_0^1 \\ &= \frac{1}{2} \left(\frac{1}{2} - \frac{1}{4} - 0 \right) = \frac{1}{8} \end{aligned}$$

51. The two bounding curves $y = x^3 - x$ and $y = x^2 + x$ intersect at the origin and at $x = 2$, with $x^2 + x > x^3 - x$ on $(0, 2)$.

Using a CAS, we find that the volume of the solid is

$$V = \int_0^2 \int_{x^3-x}^{x^2+x} (x^3 y^4 + xy^2) dy dx = \frac{13,984,735,616}{14,549,535}$$

52. For $|x| \leq 1$ and $|y| \leq 1$, $2x^2 + y^2 < 8 - x^2 - 2y^2$. Also, the cylinder is described by the inequalities $-1 \leq x \leq 1$, $-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$. So the volume is given by

$$V = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} [(8-x^2-2y^2) - (2x^2+y^2)] dy dx = \frac{13\pi}{2} \quad [\text{using a CAS}]$$

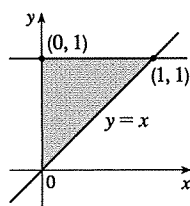
53. The two surfaces intersect in the circle $x^2 + y^2 = 1$, $z = 0$ and the region of integration is the disk D : $x^2 + y^2 \leq 1$.

Using a CAS, the volume is $\iint_D (1-x^2-y^2) dA = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1-x^2-y^2) dy dx = \frac{\pi}{2}$.

54. The projection onto the xy -plane of the intersection of the two surfaces is the circle $x^2 + y^2 = 2y \Rightarrow x^2 + y^2 - 2y = 0 \Rightarrow x^2 + (y-1)^2 = 1$, so the region of integration is given by $-1 \leq x \leq 1$, $1 - \sqrt{1-x^2} \leq y \leq 1 + \sqrt{1-x^2}$. In this region, $2y \geq x^2 + y^2$ so, using a CAS, the volume is

$$V = \int_{-1}^1 \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} [2y - (x^2 + y^2)] dy dx = \frac{\pi}{2}$$

55.

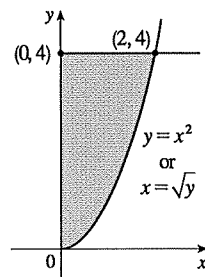


Because the region of integration is

$$D = \{(x, y) \mid 0 \leq x \leq y, 0 \leq y \leq 1\} = \{(x, y) \mid x \leq y \leq 1, 0 \leq x \leq 1\}$$

$$\text{we have } \int_0^1 \int_0^y f(x, y) dx dy = \iint_D f(x, y) dA = \int_0^1 \int_x^1 f(x, y) dy dx.$$

56.



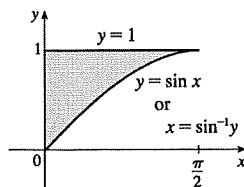
Because the region of integration is

$$D = \{(x, y) \mid x^2 \leq y \leq 4, 0 \leq x \leq 2\}$$

$$= \{(x, y) \mid 0 \leq x \leq \sqrt{y}, 0 \leq y \leq 4\}$$

$$\text{we have } \int_0^2 \int_{x^2}^4 f(x, y) dy dx = \iint_D f(x, y) dA = \int_0^4 \int_0^{\sqrt{y}} f(x, y) dx dy.$$

57.



Because the region of integration is

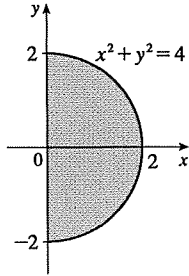
$$D = \{(x, y) \mid 0 \leq x \leq \pi/2, \sin x \leq y \leq 1\}$$

$$= \{(x, y) \mid 0 \leq x \leq \sin^{-1} y, 0 \leq y \leq 1\}$$

we have

$$\int_0^{\pi/2} \int_{\sin x}^1 f(x, y) dy dx = \iint_D f(x, y) dA = \int_0^1 \int_0^{\sin^{-1} y} f(x, y) dx dy$$

58.



Because the region of integration is

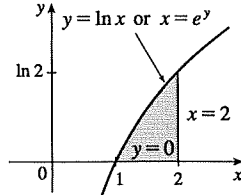
$$D = \{(x, y) \mid 0 \leq x \leq \sqrt{4-y^2}, -2 \leq y \leq 2\}$$

$$= \{(x, y) \mid -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2}, 0 \leq x \leq 2\}$$

we have

$$\int_{-2}^2 \int_0^{\sqrt{4-y^2}} f(x, y) dx dy = \iint_D f(x, y) dA = \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y) dy dx.$$

59.



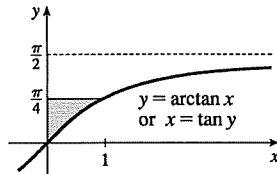
Because the region of integration is

$$D = \{(x, y) \mid 0 \leq y \leq \ln x, 1 \leq x \leq 2\} = \{(x, y) \mid e^y \leq x \leq 2, 0 \leq y \leq \ln 2\}$$

we have

$$\int_1^2 \int_0^{\ln x} f(x, y) dy dx = \iint_D f(x, y) dA = \int_0^{\ln 2} \int_{e^y}^2 f(x, y) dx dy$$

60.



Because the region of integration is

$$D = \{(x, y) \mid \arctan x \leq y \leq \frac{\pi}{4}, 0 \leq x \leq 1\}$$

$$= \{(x, y) \mid 0 \leq x \leq \tan y, 0 \leq y \leq \frac{\pi}{4}\}$$

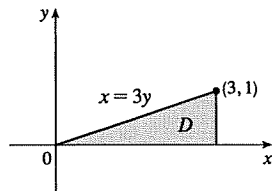
we have

$$\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y) dy dx = \iint_D f(x, y) dA = \int_0^{\pi/4} \int_0^{\tan y} f(x, y) dx dy$$

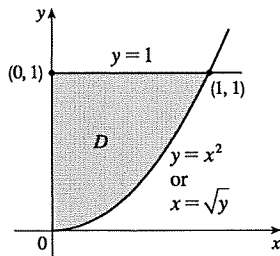
$$\int_0^1 \int_{3y}^3 e^{x^2} dx dy = \int_0^3 \int_0^{x/3} e^{x^2} dy dx = \int_0^3 [e^{x^2} y]_{y=0}^{y=x/3} dx$$

$$= \int_0^3 \left(\frac{x}{3}\right) e^{x^2} dx = \frac{1}{6} e^{x^2} \Big|_0^3 = \frac{e^9 - 1}{6}$$

61.



62.



$$\int_0^1 \int_{x^2}^1 \sqrt{y} \sin y dy dx = \int_0^1 \int_0^{\sqrt{y}} \sqrt{y} \sin y dx dy = \int_0^1 \sqrt{y} \sin y [x]_{x=0}^{x=\sqrt{y}} dy$$

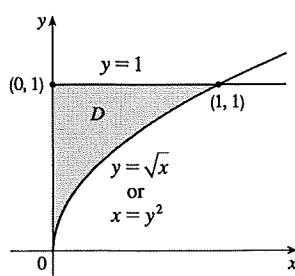
$$= \int_0^1 (\sqrt{y} \sin y) (\sqrt{y} - 0) dy = \int_0^1 y \sin y dy$$

$$= -y \cos y \Big|_0^1 + \int_0^1 \cos y dy$$

[by integrating by parts with $u = y$, $dv = \sin y dy$]

$$= [-y \cos y + \sin y]_0^1 = -\cos 1 + \sin 1 - 0 = \sin 1 - \cos 1$$

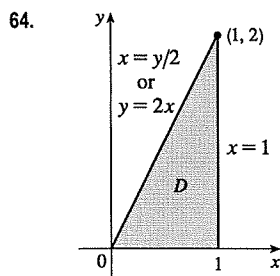
63.



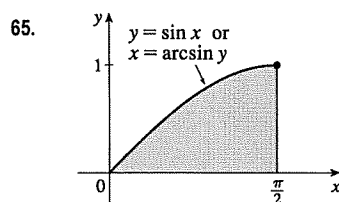
$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1} dy dx = \int_0^1 \int_0^{y^2} \sqrt{y^3 + 1} dx dy = \int_0^1 \sqrt{y^3 + 1} [x]_{x=0}^{x=y^2} dy$$

$$= \int_0^1 y^2 \sqrt{y^3 + 1} dy = \frac{2}{9} (y^3 + 1)^{3/2} \Big|_0^1$$

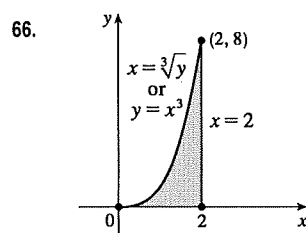
$$= \frac{2}{9} (2^{3/2} - 1^{3/2}) = \frac{2}{9} (2\sqrt{2} - 1)$$



$$\begin{aligned}
 \int_0^2 \int_{y/2}^1 y \cos(x^3 - 1) dx dy &= \int_0^1 \int_0^{2x} y \cos(x^3 - 1) dy dx \\
 &= \int_0^1 \cos(x^3 - 1) \left[\frac{1}{2} y^2 \right]_{y=0}^{y=2x} dx \\
 &= \int_0^1 2x^2 \cos(x^3 - 1) dx = \frac{2}{3} \sin(x^3 - 1) \Big|_0^1 \\
 &= \frac{2}{3} [0 - \sin(-1)] = -\frac{2}{3} \sin(-1) = \frac{2}{3} \sin 1
 \end{aligned}$$



$$\begin{aligned}
 \int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x} dx dy &= \int_0^{\pi/2} \int_0^{\sin x} \cos x \sqrt{1 + \cos^2 x} dy dx \\
 &= \int_0^{\pi/2} \cos x \sqrt{1 + \cos^2 x} [y]_{y=0}^{y=\sin x} dx \\
 &= \int_0^{\pi/2} \cos x \sqrt{1 + \cos^2 x} \sin x dx \quad \left[\begin{array}{l} \text{Let } u = \cos x, du = -\sin x dx, \\ dx = du/(-\sin x) \end{array} \right] \\
 &= \int_1^0 -u \sqrt{1 + u^2} du = -\frac{1}{3} (1 + u^2)^{3/2} \Big|_1^0 \\
 &= \frac{1}{3} (\sqrt{8} - 1) = \frac{1}{3} (2\sqrt{2} - 1)
 \end{aligned}$$



$$\begin{aligned}
 \int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy &= \int_0^2 \int_0^{x^3} e^{x^4} dy dx \\
 &= \int_0^2 e^{x^4} [y]_{y=0}^{y=x^3} dx = \int_0^2 x^3 e^{x^4} dx \\
 &= \frac{1}{4} e^{x^4} \Big|_0^2 = \frac{1}{4} (e^{16} - 1)
 \end{aligned}$$

67. $D = \{(x, y) \mid 0 \leq x \leq 1, -x + 1 \leq y \leq 1\} \cup \{(x, y) \mid -1 \leq x \leq 0, x + 1 \leq y \leq 1\}$
 $\cup \{(x, y) \mid 0 \leq x \leq 1, -1 \leq y \leq x - 1\} \cup \{(x, y) \mid -1 \leq x \leq 0, -1 \leq y \leq -x - 1\}$, all type I.

$$\begin{aligned}
 \iint_D x^2 dA &= \int_0^1 \int_{1-x}^1 x^2 dy dx + \int_{-1}^0 \int_{x+1}^1 x^2 dy dx + \int_0^1 \int_{-1}^{x-1} x^2 dy dx + \int_{-1}^0 \int_{-1}^{-x-1} x^2 dy dx \\
 &= 4 \int_0^1 \int_{1-x}^1 x^2 dy dx \quad [\text{by symmetry of the regions and because } f(x, y) = x^2 \geq 0] \\
 &= 4 \int_0^1 x^3 dx = 4 \left[\frac{1}{4} x^4 \right]_0^1 = 1
 \end{aligned}$$

68. $D = \{(x, y) \mid -1 \leq y \leq 0, -1 \leq x \leq y - y^3\} \cup \{(x, y) \mid 0 \leq y \leq 1, \sqrt{y} - 1 \leq x \leq y - y^3\}$, both type II.

$$\begin{aligned}
 \iint_D y dA &= \int_{-1}^0 \int_{-1}^{y-y^3} y dx dy + \int_0^1 \int_{\sqrt{y}-1}^{y-y^3} y dx dy = \int_{-1}^0 [xy]_{x=-1}^{x=y-y^3} dy + \int_0^1 [xy]_{x=\sqrt{y}-1}^{x=y-y^3} dy \\
 &= \int_{-1}^0 (y^2 - y^4 + y) dy + \int_0^1 (y^2 - y^4 - y^{3/2} + y) dy \\
 &= \left[\frac{1}{3} y^3 - \frac{1}{5} y^5 + \frac{1}{2} y^2 \right]_{-1}^0 + \left[\frac{1}{3} y^3 - \frac{1}{5} y^5 - \frac{2}{5} y^{5/2} + \frac{1}{2} y^2 \right]_0^1 \\
 &= \left(0 - \frac{11}{30} \right) + \left(\frac{7}{30} - 0 \right) = -\frac{2}{15}
 \end{aligned}$$