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5. $\int_1^\infty 2x^{-3}\,dx = \lim\limits_{t\to\infty}\int_1^t 2x^{-3}\,dx = \lim\limits_{t\to\infty}\left[-x^{-2}\right]_1^t = \lim\limits_{t\to\infty}\left[-t^{-2}+1\right] = 0+1 = 1.$ Convergent

6. $\displaystyle\int_{-\infty}^{-1}\frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to-\infty}\int_t^{-1}x^{-1/3}\,dx = \lim_{t\to-\infty}\left[\tfrac{3}{2}x^{2/3}\right]_t^{-1} = \lim_{t\to-\infty}\left[\tfrac{3}{2}-\tfrac{3}{2}t^{2/3}\right] = -\infty.$ Divergent

7. $\int_0^\infty e^{-2x}\,dx = \lim\limits_{t\to\infty}\int_0^t e^{-2x}\,dx = \lim\limits_{t\to\infty}\left[-\tfrac{1}{2}e^{-2x}\right]_0^t = \lim\limits_{t\to\infty}\left[-\tfrac{1}{2}e^{-2t}+\tfrac{1}{2}\right] = 0+\tfrac{1}{2} = \tfrac{1}{2}.$ Convergent

8. $\displaystyle\int_1^\infty\left(\frac{1}{3}\right)^x dx = \lim_{t\to\infty}\int_1^t 3^{-x}\,dx = \lim_{t\to\infty}\left[-\frac{3^{-x}}{\ln 3}\right]_1^t = \lim_{t\to\infty}\left[-\frac{3^{-t}}{\ln 3}+\frac{3^{-1}}{\ln 3}\right] = 0+\frac{1}{3\ln 3} = \frac{1}{\ln 27}.$ Convergent

9. $\displaystyle\int_{-2}^\infty\frac{1}{x+4}\,dx = \lim_{t\to\infty}\int_{-2}^t\frac{1}{x+4}\,dx = \lim_{t\to\infty}\Big[\ln|x+4|\Big]_{-2}^t = \lim_{t\to\infty}\Big[\ln|t+4|-\ln 2\Big] = \infty$ since $\displaystyle\lim_{t\to\infty}\ln|x+4| = \infty.$

Divergent

10. $\displaystyle\int_1^\infty\frac{1}{x^2+4}\,dx = \lim_{t\to\infty}\int_1^t\frac{1}{x^2+4}\,dx = \lim_{t\to\infty}\left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_1^t = \lim_{t\to\infty}\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)-\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)\right]$

$\displaystyle = \frac{\pi}{4}-\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right).$ Convergent

11. $\displaystyle\int_3^\infty\frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty}\int_3^t(x-2)^{-3/2}\,dx = \lim_{t\to\infty}\left[-2\,(x-2)^{-1/2}\right]_3^t$ $[u = x-2,\ du = dx]$

$\displaystyle = \lim_{t\to\infty}\left(\frac{-2}{\sqrt{t-2}}+\frac{2}{\sqrt{1}}\right) = 0+2 = 2.$ Convergent

12. $\displaystyle\int_0^\infty\frac{1}{\sqrt[4]{1+x}}\,dx = \lim_{t\to\infty}\int_0^t(1+x)^{-1/4}\,dx = \lim_{t\to\infty}\left[\tfrac{4}{3}(1+x)^{3/4}\right]_0^t$ $[u = 1+x,\ du = dx]$

$\displaystyle = \lim_{t\to\infty}\left[\tfrac{4}{3}(1+t)^{3/4}-\tfrac{4}{3}\right] = \infty.$ Divergent

13. $\displaystyle\int_{-\infty}^0\frac{x}{(x^2+1)^3}\,dx = \lim_{t\to-\infty}\int_t^0\frac{x}{(x^2+1)^3}\,dx = \lim_{t\to-\infty}\left[-\frac{1}{4}(x^2+1)^{-2}\right]_t^0 = \lim_{t\to-\infty}\left[-\frac{1}{4\,(x^2+1)^2}\right]_t^0$

$\displaystyle = \lim_{t\to-\infty}\left[-\frac{1}{4}+\frac{1}{4(t^2+1)^2}\right] = -\frac{1}{4}+0 = -\frac{1}{4}.$ Convergent

14. $\displaystyle\int_{-\infty}^{-3}\frac{x}{4-x^2}\,dx = \lim_{t\to-\infty}\int_t^{-3}\frac{x}{4-x^2}\,dx = \lim_{t\to-\infty}\left[-\frac{1}{2}\ln|4-x^2|\right]_t^{-3}$

$\displaystyle = \lim_{t\to-\infty}\left[-\tfrac{1}{2}\ln 5+\tfrac{1}{2}\ln|4-t^2|\right] = \infty$ since $\displaystyle\lim_{t\to-\infty}\ln|4-t^2| = \infty.$ Divergent

15. $\displaystyle\int_1^\infty\frac{x^2+x+1}{x^4}\,dx = \lim_{t\to\infty}\int_1^t(x^{-2}+x^{-3}+x^{-4})\,dx$

$\displaystyle = \lim_{t\to\infty}\left[-x^{-1}-\frac{1}{2}x^{-2}-\frac{1}{3}x^{-3}\right]_1^t = \lim_{t\to\infty}\left[-\frac{1}{x}-\frac{1}{2x^2}-\frac{1}{3x^3}\right]_1^t$

$\displaystyle = \lim_{t\to\infty}\left[\left(-\frac{1}{t}-\frac{1}{2t^2}-\frac{1}{3t^3}\right)-\left(-1-\frac{1}{2}-\frac{1}{3}\right)\right] = 0+\frac{11}{6} = \frac{11}{6}.$ Convergent

16. $\displaystyle\int_2^\infty\frac{x}{\sqrt{x^2-1}}\,dx = \lim_{t\to\infty}\int_2^t\frac{x}{\sqrt{x^2-1}}\,dx = \lim_{t\to\infty}\left[\sqrt{x^2-1}\right]_2^t = \lim_{t\to\infty}\left[\sqrt{t^2-1}-\sqrt{3}\right] = \infty.$ Divergent

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17. $\displaystyle\int_0^\infty \frac{e^x}{(1+e^x)^2}\,dx = \lim_{t\to\infty}\int_0^t \frac{e^x}{(1+e^x)^2}\,dx = \lim_{t\to\infty}\Big[-(1+e^x)^{-1}\Big]_0^t = \lim_{t\to\infty}\left[-\frac{1}{1+e^x}\right]_0^t$

$\displaystyle= \lim_{t\to\infty}\left[-\frac{1}{1+e^t}+\frac{1}{2}\right] = 0+\frac{1}{2} = \frac{1}{2}.$ Convergent

18. $\displaystyle I = \int_{-\infty}^{-1}\frac{x^2+x}{x^3}\,dx = \int_{-\infty}^{-1}\frac{1}{x}\,dx + \int_{-\infty}^{-1}\frac{1}{x^2}\,dx = I_1 + I_2.$

Now, $\displaystyle\int_{-\infty}^{-1}\frac{1}{x}\,dx = \lim_{t\to-\infty}\int_t^{-1}\frac{1}{x}\,dx = \lim_{t\to-\infty}\Big[\ln|x|\Big]_t^{-1} = \lim_{t\to-\infty}\Big[\ln 1 - \ln|t|\Big] = -\infty.$

Since I_1 is divergent, I is divergent. Divergent

19. $\int_{-\infty}^\infty xe^{-x^2}\,dx = \int_{-\infty}^0 xe^{-x^2}\,dx + \int_0^\infty xe^{-x^2}\,dx.$

$\int_{-\infty}^0 xe^{-x^2}\,dx = \lim_{t\to-\infty}\left(-\frac{1}{2}\right)\Big[e^{-x^2}\Big]_t^0 = \lim_{t\to-\infty}\left(-\frac{1}{2}\right)\left(1-e^{-t^2}\right) = -\frac{1}{2}\cdot(1-0) = -\frac{1}{2}$, and

$\int_0^\infty xe^{-x^2}\,dx = \lim_{t\to\infty}\left(-\frac{1}{2}\right)\Big[e^{-x^2}\Big]_0^t = \lim_{t\to\infty}\left(-\frac{1}{2}\right)\left(e^{-t^2}-1\right) = -\frac{1}{2}\cdot(0-1) = \frac{1}{2}.$

Therefore, $\int_{-\infty}^\infty xe^{-x^2}\,dx = -\frac{1}{2}+\frac{1}{2} = 0.$ Convergent

20. $\displaystyle I = \int_{-\infty}^\infty \frac{x}{x^2+1}\,dx = \int_{-\infty}^0 \frac{x}{x^2+1}\,dx + \int_0^\infty \frac{x}{x^2+1}\,dx = I_1 + I_2$, but

$\displaystyle I_2 = \lim_{t\to\infty}\int_0^t \frac{x}{x^2+1}\,dx = \lim_{t\to\infty}\int_1^{t^2+1}\frac{1/2}{u}\,du \quad \begin{bmatrix} u = x^2+1, \\ du = 2x\,dx \end{bmatrix} \quad = \frac{1}{2}\lim_{t\to\infty}\Big[\ln|u|\Big]_1^{t^2+1}$

$\displaystyle= \frac{1}{2}\lim_{t\to\infty}\Big[\ln|t^2+1| - 0\Big] = \infty$

Since I_2 is divergent, I is divergent, and there is no need to evaluate I_1. Divergent

21. $I = \int_{-\infty}^\infty \cos 2t\,dt = \int_{-\infty}^0 \cos 2t\,dt + \int_0^\infty \cos 2t\,dt = I_1 + I_2$, but $I_1 = \lim_{s\to-\infty}\Big[\frac{1}{2}\sin 2t\Big]_s^0 = \lim_{s\to-\infty}\left(-\frac{1}{2}\sin 2s\right)$, and this limit does not exist. Since I_1 is divergent, I is divergent, and there is no need to evaluate I_2. Divergent

22. $\displaystyle\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty}\Big[e^{-1/x}\Big]_1^t = \lim_{t\to\infty}\left(e^{-1/t}-e^{-1}\right) = 1-\frac{1}{e}.$ Convergent

23. $\int_0^\infty \sin^2\alpha\,d\alpha = \lim_{t\to\infty}\int_0^t \frac{1}{2}(1-\cos 2\alpha)\,d\alpha = \lim_{t\to\infty}\Big[\frac{1}{2}\left(\alpha-\frac{1}{2}\sin 2\alpha\right)\Big]_0^t = \lim_{t\to\infty}\Big[\frac{1}{2}\left(t-\frac{1}{2}\sin 2t\right)-0\Big] = \infty.$

Divergent

24. $\int_0^\infty \sin\theta\, e^{\cos\theta}\,d\theta = \lim_{t\to\infty}\int_0^t \sin\theta\, e^{\cos\theta}\,d\theta = \lim_{t\to\infty}\Big[-e^{\cos\theta}\Big]_0^t = \lim_{t\to\infty}\left(-e^{\cos t}+e\right)$

This limit does not exist since $\cos t$ oscillates in value between -1 and 1, so $e^{\cos t}$ oscillates in value between e^{-1} and e^1. Divergent

25. $\displaystyle\int_1^\infty \frac{1}{x^2+x}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x(x+1)}\,dx = \lim_{t\to\infty}\int_1^t\left(\frac{1}{x}-\frac{1}{x+1}\right)dx$ [partial fractions]

$\displaystyle= \lim_{t\to\infty}\Big[\ln|x| - \ln|x+1|\Big]_1^t = \lim_{t\to\infty}\left[\ln\left|\frac{x}{x+1}\right|\right]_1^t = \lim_{t\to\infty}\left(\ln\frac{t}{t+1}-\ln\frac{1}{2}\right) = 0-\ln\frac{1}{2} = \ln 2.$

Convergent

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