HW # 4, SECTION 11.5 SOLUTIONS Sec 11.5

3. $-\frac{2}{5}+\frac{4}{6}-\frac{6}{7}+\frac{8}{8}-\frac{10}{9}+\cdots=\sum_{n=1}^{\infty}(-1)^n\frac{2n}{n+4}$. Now $\lim_{n\to\infty} b_n=\lim_{n\to\infty}\frac{2n}{n+4}=\lim_{n\to\infty}\frac{2}{1+4/n}=\frac{2}{1}\neq 0$. Since $\lim_{n\to\infty} a_n\neq 0$ (in fact the limit does not exist), the series diverges by the Test for Divergence.

4. $\frac{1}{\ln 3}-\frac{1}{\ln 4}+\frac{1}{\ln 5}-\frac{1}{\ln 6}+\frac{1}{\ln 7}-\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\ln(n+2)}$. Now $b_n=\frac{1}{\ln(n+2)}>0$, $\{b_n\}$ is decreasing, and $\lim_{n\to\infty} b_n=0$, so the series converges by the Alternating Series Test.

5. $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{3+5n}=\sum_{n=1}^{\infty}(-1)^{n-1}b_n$. Now $b_n=\frac{1}{3+5n}>0$, $\{b_n\}$ is decreasing, and $\lim_{n\to\infty} b_n=0$, so the series converges by the Alternating Series Test.

6. $\sum_{n=0}^{\infty}a_n=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n+1}}=\sum_{n=0}^{\infty}(-1)^{n+1}b_n$. Now $b_n=\frac{1}{\sqrt{n+1}}>0$, $\{b_n\}$ is decreasing, and $\lim_{n\to\infty} b_n=0$, so the series converges by the Alternating Series Test.

7. $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}(-1)^n\frac{3n-1}{2n+1}=\sum_{n=1}^{\infty}(-1)^n b_n$. Now $\lim_{n\to\infty} b_n=\lim_{n\to\infty}\frac{3-1/n}{2+1/n}=\frac{3}{2}\neq 0$. Since $\lim_{n\to\infty} a_n\neq 0$ (in fact the limit does not exist), the series diverges by the Test for Divergence.

8. $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}(-1)^n\frac{n^2}{n^2+n+1}=\sum_{n=1}^{\infty}(-1)^n b_n$. Now $\lim_{n\to\infty} b_n=\lim_{n\to\infty}\frac{n^2}{n^2+n+1}=\lim_{n\to\infty}\frac{1}{1+1/n+1/n^2}=1\neq 0$. Since $\lim_{n\to\infty} a_n\neq 0$, the series diverges by the Test for Divergence.

9. $\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}(-1)^n e^{-n}=\sum_{n=1}^{\infty}(-1)^n b_n$. Now $b_n=\frac{1}{e^n}>0$, $\{b_n\}$ is decreasing, and $\lim_{n\to\infty} b_n=0$, so the series converges by the Alternating Series Test.

10. $b_n=\frac{\sqrt{n}}{2n+3}>0$ for $n\geq 1$. $\{b_n\}$ is decreasing for $n\geq 2$ since

$$\left(\frac{\sqrt{x}}{2x+3}\right)'=\frac{(2x+3)\left(\frac{1}{2}x^{-1/2}\right)-x^{1/2}(2)}{(2x+3)^2}=\frac{\frac{1}{2}x^{-1/2}[(2x+3)-4x]}{(2x+3)^2}=\frac{3-2x}{2\sqrt{x}\,(2x+3)^2}<0 \text{ for } x>\tfrac{3}{2}.$$

Also, $\lim_{n\to\infty} b_n=\lim_{n\to\infty}\frac{\sqrt{n}/\sqrt{n}}{(2n+3)/\sqrt{n}}=\lim_{n\to\infty}\frac{1}{2\sqrt{n}+3/\sqrt{n}}=0$. Thus, the series $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{2n+3}$ converges by the Alternating Series Test.

11. $b_n=\frac{n^2}{n^3+4}>0$ for $n\geq 1$. $\{b_n\}$ is decreasing for $n\geq 2$ since

$$\left(\frac{x^2}{x^3+4}\right)'=\frac{(x^3+4)(2x)-x^2(3x^2)}{(x^3+4)^2}=\frac{x(2x^3+8-3x^3)}{(x^3+4)^2}=\frac{x(8-x^3)}{(x^3+4)^2}<0 \text{ for } x>2. \text{ Also,}$$

$\lim_{n\to\infty} b_n=\lim_{n\to\infty}\frac{1/n}{1+4/n^3}=0$. Thus, the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{n^3+4}$ converges by the Alternating Series Test.