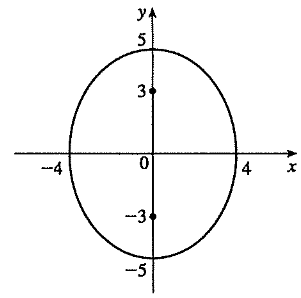
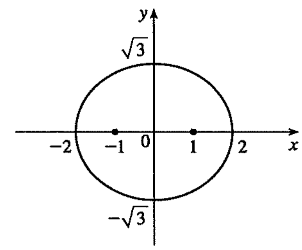
HW # 7, PART VI - B, Section 10.5 Solutions (ON ELLIPSES)

SEC. 10.5

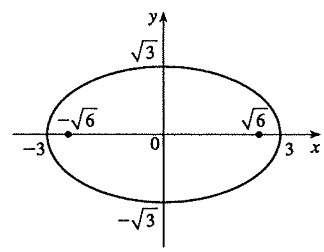
11. $\dfrac{x^2}{16}+\dfrac{y^2}{25}=1 \;\Rightarrow\; a=\sqrt{25}=5,\, b=\sqrt{16}=4,$ $c=\sqrt{a^2-b^2}=\sqrt{25-16}=\sqrt{9}=3$. The ellipse is centered at $(0,0)$ with vertices $(0,\pm5)$. The foci are $(0,\pm3)$.



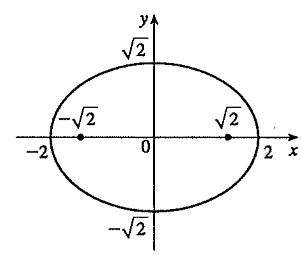
12. $\dfrac{x^2}{4}+\dfrac{y^2}{3}=1 \;\Rightarrow\; a=\sqrt{4}=2,\, b=\sqrt{3},$ $c=\sqrt{a^2-b^2}=\sqrt{4-3}=\sqrt{1}=1$. The ellipse is centered at $(0,0)$ with vertices $(\pm2,0)$. The foci are $(\pm1,0)$.



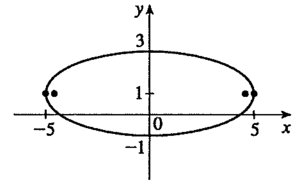
13. $x^2+3y^2=9 \;\Leftrightarrow\; \dfrac{x^2}{9}+\dfrac{y^2}{3}=1 \;\Rightarrow\; a=\sqrt{9}=3,\, b=\sqrt{3},$ $c=\sqrt{a^2-b^2}=\sqrt{9-3}=\sqrt{6}$. The ellipse is centered at $(0,0)$ with vertices $(\pm3,0)$. The foci are $(\pm\sqrt{6},0)$.



14. $x^2=4-2y^2 \;\Leftrightarrow\; x^2+2y^2=4 \;\Leftrightarrow\; \dfrac{x^2}{4}+\dfrac{y^2}{2}=1 \;\Rightarrow$ $a=\sqrt{4}=2,\, b=\sqrt{2},\, c=\sqrt{a^2-b^2}=\sqrt{4-2}=\sqrt{2}$. The ellipse is centered at $(0,0)$ with vertices $(\pm2,0)$. The foci are $(\pm\sqrt{2},0)$.



15. $4x^2+25y^2-50y=75 \;\Leftrightarrow\; 4x^2+25(y^2-2y+1)=75+25 \;\Leftrightarrow$ $4x^2+25(y-1)^2=100 \;\Leftrightarrow\; \dfrac{x^2}{25}+\dfrac{(y-1)^2}{4}=1 \;\Rightarrow\; a=\sqrt{25}=5,$ $b=\sqrt{4}=2,\, c=\sqrt{25-4}=\sqrt{21}$. The ellipse is centered at $(0,1)$ with vertices $(\pm5,1)$. The foci are $(\pm\sqrt{21},1)$.

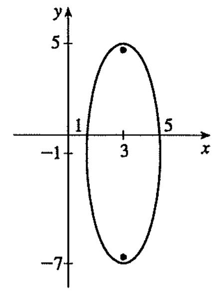


16. $9x^2 - 54x + y^2 + 2y + 46 = 0 \quad \Leftrightarrow$

$9(x^2 - 6x + 9) + y^2 + 2y + 1 = -46 + 81 + 1 \quad \Leftrightarrow$

$9(x-3)^2 + (y+1)^2 = 36 \quad \Leftrightarrow \quad \dfrac{(x-3)^2}{4} + \dfrac{(y+1)^2}{36} = 1 \quad \Rightarrow$

$a = \sqrt{36} = 6$, $b = \sqrt{4} = 2$, $c = \sqrt{36-4} = \sqrt{32} = 4\sqrt{2}$. The ellipse is centered at $(3, -1)$ with vertices $(3, 5)$ and $(3, -7)$. The foci are $\left(3, -1 \pm 4\sqrt{2}\right)$.



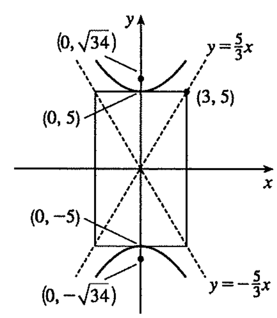
17. The center is $(0, 0)$, $a = 3$, and $b = 2$, so an equation is $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$. $c = \sqrt{a^2 - b^2} = \sqrt{5}$, so the foci are $\left(0, \pm\sqrt{5}\right)$.

18. The ellipse is centered at $(2, 1)$, with $a = 3$ and $b = 2$. An equation is $\dfrac{(x-2)^2}{9} + \dfrac{(y-1)^2}{4} = 1$. $c = \sqrt{a^2 - b^2} = \sqrt{5}$, so the foci are $\left(2 \pm \sqrt{5}, 1\right)$.

19. $\dfrac{y^2}{25} - \dfrac{x^2}{9} = 1 \quad \Rightarrow \quad a = 5, b = 3, c = \sqrt{a^2 + b^2} = \sqrt{25 + 9} = \sqrt{34} \quad \Rightarrow$

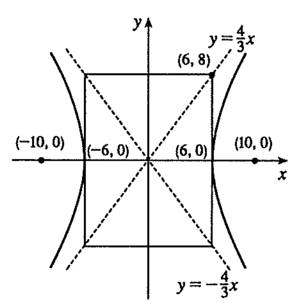
center $(0, 0)$, vertices $(0, \pm 5)$, foci $\left(0, \pm\sqrt{34}\right)$, asymptotes $y = \pm\frac{5}{3}x$.

Note: It is helpful to draw a $2a$-by-$2b$ rectangle whose center is the center of the hyperbola. The asymptotes are the extended diagonals of the rectangle.



20. $\dfrac{x^2}{36} - \dfrac{y^2}{64} = 1 \quad \Rightarrow \quad a = 6, b = 8, c = \sqrt{a^2 + b^2} = \sqrt{36 + 64} = 10 \quad \Rightarrow$

center $(0, 0)$, vertices $(\pm 6, 0)$, foci $(\pm 10, 0)$, asymptotes $y = \pm\frac{8}{6}x = \pm\frac{4}{3}x$



21. $x^2 - y^2 = 100 \quad \Leftrightarrow \quad \dfrac{x^2}{100} - \dfrac{y^2}{100} = 1 \quad \Rightarrow \quad a = b = 10$,

$c = \sqrt{100 + 100} = 10\sqrt{2} \quad \Rightarrow \quad$ center $(0, 0)$, vertices $(\pm 10, 0)$,

foci $\left(\pm 10\sqrt{2}, 0\right)$, asymptotes $y = \pm\frac{10}{10}x = \pm x$