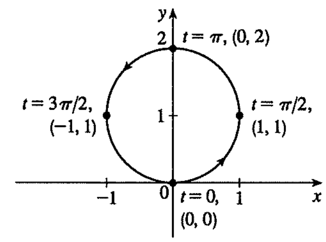
HW #7, Section 10.1 Solutions

10. $x = \sin t, \quad y = 1 - \cos t, \quad 0 \le t \le 2\pi$

(a)

t	0	$\pi/2$	π	$3\pi/2$	2π
x	0	1	0	-1	0
y	0	1	2	1	0



(b) $x = \sin t, y = 1 - \cos t \quad [\text{or } y - 1 = -\cos t] \quad \Rightarrow$

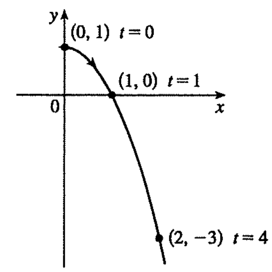
$x^2 + (y-1)^2 = (\sin t)^2 + (-\cos t)^2 \quad \Rightarrow \quad x^2 + (y-1)^2 = 1.$

As t varies from 0 to 2π, the circle with center $(0, 1)$ and radius 1 is traced out.

11. $x = \sqrt{t}, \; y = 1 - t$

(a)

t	0	1	2	3	4
x	0	1	1.414	1.732	2
y	1	0	-1	-2	-3



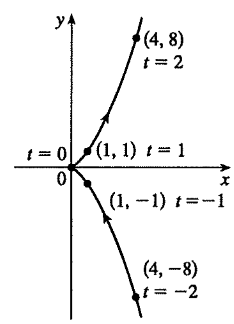
(b) $x = \sqrt{t} \quad \Rightarrow \quad t = x^2 \quad \Rightarrow \quad y = 1 - t = 1 - x^2$. Since $t \ge 0$, $x \ge 0$.

So the curve is the right half of the parabola $y = 1 - x^2$.

12. $x = t^2, \quad y = t^3$

(a)

t	-2	-1	0	1	2
x	4	1	0	1	4
y	-8	-1	0	1	8

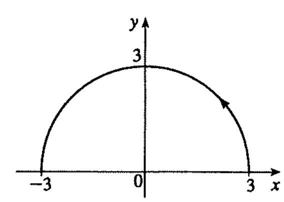


(b) $y = t^3 \quad \Rightarrow \quad t = \sqrt[3]{y} \quad \Rightarrow \quad x = t^2 = \left(\sqrt[3]{y}\right)^2 = y^{2/3}. \quad t \in \mathbb{R}, y \in \mathbb{R}, x \ge 0.$

13. (a) $x = 3\cos t, \quad y = 3\sin t, \quad 0 \le t \le \pi$

$x^2 + y^2 = 9\cos^2 t + 9\sin^2 t = 9(\cos^2 t + \sin^2 t) = 9$, which is the equation of a circle with radius 3. For $0 \le t \le \pi/2$, we have $3 \ge x \ge 0$ and $0 \le y \le 3$. For $\pi/2 < t \le \pi$, we have $0 > x \ge -3$ and $3 > y \ge 0$. Thus, the curve is the top half of the circle $x^2 + y^2 = 9$ traced counterclockwise.

(b)



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27. $x = 5\sin t,\, y = 2\cos t \;\Rightarrow\; \sin t = \dfrac{x}{5},\, \cos t = \dfrac{y}{2}.\;\; \sin^2 t + \cos^2 t = 1 \;\Rightarrow\; \left(\dfrac{x}{5}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$. The motion of the particle takes place on an ellipse centered at $(0,0)$. As t goes from $-\pi$ to 5π, the particle starts at the point $(0,-2)$ and moves clockwise around the ellipse 3 times.

28. $y = \cos^2 t = 1 - \sin^2 t = 1 - x^2$. The motion of the particle takes place on the parabola $y = 1 - x^2$. As t goes from -2π to $-\pi$, the particle starts at the point $(0,1)$, moves to $(1,0)$, and goes back to $(0,1)$. As t goes from $-\pi$ to 0, the particle moves to $(-1,0)$ and goes back to $(0,1)$. The particle repeats this motion as t goes from 0 to 2π.

29. We must have $1 \le x \le 4$ and $2 \le y \le 3$. So the graph of the curve must be contained in the rectangle $[1,4]$ by $[2,3]$.

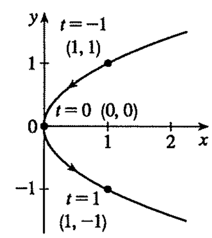
30. (a) From the first graph, we have $1 \le x \le 2$. From the second graph, we have $-1 \le y \le 1$. The only choice that satisfies either of those conditions is III.

(b) From the first graph, the values of x cycle through the values from -2 to 2 four times. From the second graph, the values of y cycle through the values from -2 to 2 six times. Choice I satisfies these conditions.

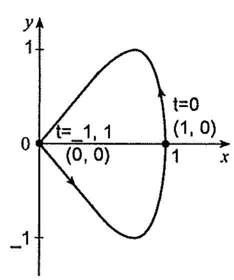
(c) From the first graph, the values of x cycle through the values from -2 to 2 three times. From the second graph, we have $0 \le y \le 2$. Choice IV satisfies these conditions.

(d) From the first graph, the values of x cycle through the values from -2 to 2 two times. From the second graph, the values of y do the same thing. Choice II satisfies these conditions.

31. When $t = -1$, $(x,y) = (1,1)$. As t increases to 0, x and y both decrease to 0. As t increases from 0 to 1, x increases from 0 to 1 and y decreases from 0 to -1. As t increases beyond 1, x continues to increase and y continues to decrease. For $t < -1$, x and y are both positive and decreasing. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.



32. When $t = -1$, $(x,y) = (0,0)$. As t increases to 0, x increases from 0 to 1, while y first decreases to -1 and then increases to 0. As t increases from 0 to 1, x decreases from 1 to 0, while y first increases to 1 and then decreases to 0. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.



33. When $t = -1$, $(x,y) = (0,1)$. As t increases to 0, x increases from 0 to 1 and y decreases from 1 to 0. As t increases from 0 to 1, the curve is retraced in the opposite direction with x decreasing from 1 to 0 and y increasing from 0 to 1. We could achieve greater accuracy by estimating x- and y-values for selected values of t from the given graphs and plotting the corresponding points.