HW #3 Section 7.8 SOLUTIONS

5. $\int_1^\infty 2x^{-3}\,dx = \lim\limits_{t\to\infty}\int_1^t 2x^{-3}\,dx = \lim\limits_{t\to\infty}\Big[-x^{-2}\Big]_1^t = \lim\limits_{t\to\infty}\Big[-t^{-2}+1\Big] = 0+1=1.$ Convergent

6. $\displaystyle\int_{-\infty}^{-1}\frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to-\infty}\int_t^{-1}x^{-1/3}\,dx = \lim_{t\to-\infty}\Big[\tfrac{3}{2}x^{2/3}\Big]_t^{-1} = \lim_{t\to-\infty}\Big[\tfrac{3}{2}-\tfrac{3}{2}t^{2/3}\Big] = -\infty.$ Divergent

7. $\int_0^\infty e^{-2x}\,dx = \lim\limits_{t\to\infty}\int_0^t e^{-2x}\,dx = \lim\limits_{t\to\infty}\Big[-\tfrac{1}{2}e^{-2x}\Big]_0^t = \lim\limits_{t\to\infty}\Big[-\tfrac{1}{2}e^{-2t}+\tfrac{1}{2}\Big] = 0+\tfrac{1}{2}=\tfrac{1}{2}.$ Convergent

8. $\displaystyle\int_1^\infty\left(\frac{1}{3}\right)^x dx = \lim_{t\to\infty}\int_1^t 3^{-x}\,dx = \lim_{t\to\infty}\left[-\frac{3^{-x}}{\ln 3}\right]_1^t = \lim_{t\to\infty}\left[-\frac{3^{-t}}{\ln 3}+\frac{3^{-1}}{\ln 3}\right] = 0+\frac{1}{3\ln 3}=\frac{1}{\ln 27}.$ Convergent

9. $\displaystyle\int_{-2}^\infty\frac{1}{x+4}\,dx = \lim_{t\to\infty}\int_{-2}^t\frac{1}{x+4}\,dx = \lim_{t\to\infty}\Big[\ln|x+4|\Big]_{-2}^t = \lim_{t\to\infty}\Big[\ln|t+4|-\ln 2\Big] = \infty$ since $\lim\limits_{t\to\infty}\ln|x+4|=\infty.$

Divergent

10. $\displaystyle\int_1^\infty\frac{1}{x^2+4}\,dx = \lim_{t\to\infty}\int_1^t\frac{1}{x^2+4}\,dx = \lim_{t\to\infty}\left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_1^t = \lim_{t\to\infty}\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)-\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)\right]$

$\displaystyle= \frac{\pi}{4}-\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right).$ Convergent

11. $\displaystyle\int_3^\infty\frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty}\int_3^t(x-2)^{-3/2}\,dx = \lim_{t\to\infty}\Big[-2\,(x-2)^{-1/2}\Big]_3^t$ $[u=x-2,\ du=dx]$

$\displaystyle= \lim_{t\to\infty}\left(\frac{-2}{\sqrt{t-2}}+\frac{2}{\sqrt{1}}\right) = 0+2=2.$ Convergent

12. $\displaystyle\int_0^\infty\frac{1}{\sqrt[4]{1+x}}\,dx = \lim_{t\to\infty}\int_0^t(1+x)^{-1/4}\,dx = \lim_{t\to\infty}\Big[\tfrac{4}{3}(1+x)^{3/4}\Big]_0^t$ $[u=1+x,\ du=dx]$

$\displaystyle= \lim_{t\to\infty}\Big[\tfrac{4}{3}(1+t)^{3/4}-\tfrac{4}{3}\Big] = \infty.$ Divergent

13. $\displaystyle\int_{-\infty}^0\frac{x}{(x^2+1)^3}\,dx = \lim_{t\to-\infty}\int_t^0\frac{x}{(x^2+1)^3}\,dx = \lim_{t\to-\infty}\left[-\frac{1}{4}(x^2+1)^{-2}\right]_t^0 = \lim_{t\to-\infty}\left[-\frac{1}{4\,(x^2+1)^2}\right]_t^0$

$\displaystyle= \lim_{t\to-\infty}\left[-\frac{1}{4}+\frac{1}{4(t^2+1)^2}\right] = -\frac{1}{4}+0=-\frac{1}{4}.$ Convergent

14. $\displaystyle\int_{-\infty}^{-3}\frac{x}{4-x^2}\,dx = \lim_{t\to-\infty}\int_t^{-3}\frac{x}{4-x^2}\,dx = \lim_{t\to-\infty}\left[-\frac{1}{2}\ln|4-x^2|\right]_t^{-3}$

$\displaystyle= \lim_{t\to-\infty}\left[-\tfrac{1}{2}\ln 5+\tfrac{1}{2}\ln|4-t^2|\right] = \infty$ since $\lim\limits_{t\to-\infty}\ln|4-t^2|=\infty.$ Divergent

15. $\displaystyle\int_1^\infty\frac{x^2+x+1}{x^4}\,dx = \lim_{t\to\infty}\int_1^t(x^{-2}+x^{-3}+x^{-4})\,dx$

$\displaystyle= \lim_{t\to\infty}\left[-x^{-1}-\frac{1}{2}x^{-2}-\frac{1}{3}x^{-3}\right]_1^t = \lim_{t\to\infty}\left[-\frac{1}{x}-\frac{1}{2x^2}-\frac{1}{3x^3}\right]_1^t$

$\displaystyle= \lim_{t\to\infty}\left[\left(-\frac{1}{t}-\frac{1}{2t^2}-\frac{1}{3t^3}\right)-\left(-1-\frac{1}{2}-\frac{1}{3}\right)\right] = 0+\frac{11}{6}=\frac{11}{6}.$ Convergent

16. $\displaystyle\int_2^\infty\frac{x}{\sqrt{x^2-1}}\,dx = \lim_{t\to\infty}\int_2^t\frac{x}{\sqrt{x^2-1}}\,dx = \lim_{t\to\infty}\Big[\sqrt{x^2-1}\Big]_2^t = \lim_{t\to\infty}\Big[\sqrt{t^2-1}-\sqrt{3}\Big] = \infty.$ Divergent

Sec 7.8

17. $\displaystyle\int_0^\infty \frac{e^x}{(1+e^x)^2}\,dx = \lim_{t\to\infty}\int_0^t \frac{e^x}{(1+e^x)^2}\,dx = \lim_{t\to\infty}\Big[-(1+e^x)^{-1}\Big]_0^t = \lim_{t\to\infty}\left[-\frac{1}{1+e^x}\right]_0^t$

$\displaystyle = \lim_{t\to\infty}\left[-\frac{1}{1+e^t}+\frac{1}{2}\right] = 0+\frac{1}{2} = \frac{1}{2}.$ Convergent

18. $\displaystyle I = \int_{-\infty}^{-1}\frac{x^2+x}{x^3}\,dx = \int_{-\infty}^{-1}\frac{1}{x}\,dx + \int_{-\infty}^{-1}\frac{1}{x^2}\,dx = I_1 + I_2.$

Now, $\displaystyle\int_{-\infty}^{-1}\frac{1}{x}\,dx = \lim_{t\to-\infty}\int_t^{-1}\frac{1}{x}\,dx = \lim_{t\to-\infty}\Big[\ln|x|\Big]_t^{-1} = \lim_{t\to-\infty}\Big[\ln 1 - \ln|t|\Big] = -\infty.$

Since I_1 is divergent, I is divergent. Divergent

19. $\int_{-\infty}^{\infty} xe^{-x^2}\,dx = \int_{-\infty}^{0} xe^{-x^2}\,dx + \int_0^\infty xe^{-x^2}\,dx.$

$\int_{-\infty}^{0} xe^{-x^2}\,dx = \lim_{t\to-\infty}\left(-\frac{1}{2}\right)\Big[e^{-x^2}\Big]_t^0 = \lim_{t\to-\infty}\left(-\frac{1}{2}\right)\left(1-e^{-t^2}\right) = -\frac{1}{2}\cdot(1-0) = -\frac{1}{2}$, and

$\int_0^\infty xe^{-x^2}\,dx = \lim_{t\to\infty}\left(-\frac{1}{2}\right)\Big[e^{-x^2}\Big]_0^t = \lim_{t\to\infty}\left(-\frac{1}{2}\right)\left(e^{-t^2}-1\right) = -\frac{1}{2}\cdot(0-1) = \frac{1}{2}.$

Therefore, $\int_{-\infty}^{\infty} xe^{-x^2}\,dx = -\frac{1}{2}+\frac{1}{2} = 0.$ Convergent

20. $\displaystyle I = \int_{-\infty}^{\infty}\frac{x}{x^2+1}\,dx = \int_{-\infty}^{0}\frac{x}{x^2+1}\,dx + \int_0^\infty \frac{x}{x^2+1}\,dx = I_1 + I_2$, but

$\displaystyle I_2 = \lim_{t\to\infty}\int_0^t \frac{x}{x^2+1}\,dx = \lim_{t\to\infty}\int_1^{t^2+1}\frac{1/2}{u}\,du \quad \left[\begin{matrix} u = x^2+1, \\ du = 2x\,dx \end{matrix}\right] \quad = \frac{1}{2}\lim_{t\to\infty}\Big[\ln|u|\Big]_1^{t^2+1}$

$\displaystyle = \frac{1}{2}\lim_{t\to\infty}\Big[\ln|t^2+1| - 0\Big] = \infty$

Since I_2 is divergent, I is divergent, and there is no need to evaluate I_1. Divergent

21. $I = \int_{-\infty}^{\infty}\cos 2t\,dt = \int_{-\infty}^{0}\cos 2t\,dt + \int_0^\infty \cos 2t\,dt = I_1 + I_2$, but $I_1 = \lim_{s\to-\infty}\Big[\frac{1}{2}\sin 2t\Big]_s^0 = \lim_{s\to-\infty}\left(-\frac{1}{2}\sin 2s\right)$, and this limit does not exist. Since I_1 is divergent, I is divergent, and there is no need to evaluate I_2. Divergent

22. $\displaystyle\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{e^{-1/x}}{x^2}\,dx = \lim_{t\to\infty}\Big[e^{-1/x}\Big]_1^t = \lim_{t\to\infty}\left(e^{-1/t}-e^{-1}\right) = 1-\frac{1}{e}.$ Convergent

23. $\int_0^\infty \sin^2\alpha\,d\alpha = \lim_{t\to\infty}\int_0^t \frac{1}{2}(1-\cos 2\alpha)\,d\alpha = \lim_{t\to\infty}\left[\frac{1}{2}\left(\alpha - \frac{1}{2}\sin 2\alpha\right)\right]_0^t = \lim_{t\to\infty}\left[\frac{1}{2}\left(t-\frac{1}{2}\sin 2t\right) - 0\right] = \infty.$

Divergent

24. $\int_0^\infty \sin\theta\, e^{\cos\theta}\,d\theta = \lim_{t\to\infty}\int_0^t \sin\theta\, e^{\cos\theta}\,d\theta = \lim_{t\to\infty}\Big[-e^{\cos\theta}\Big]_0^t = \lim_{t\to\infty}\left(-e^{\cos t}+e\right)$

This limit does not exist since $\cos t$ oscillates in value between -1 and 1, so $e^{\cos t}$ oscillates in value between e^{-1} and e^1. Divergent

25. $\displaystyle\int_1^\infty \frac{1}{x^2+x}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x(x+1)}\,dx = \lim_{t\to\infty}\int_1^t\left(\frac{1}{x}-\frac{1}{x+1}\right)dx$ [partial fractions]

$\displaystyle = \lim_{t\to\infty}\Big[\ln|x| - \ln|x+1|\Big]_1^t = \lim_{t\to\infty}\left[\ln\left|\frac{x}{x+1}\right|\right]_1^t = \lim_{t\to\infty}\left(\ln\frac{t}{t+1} - \ln\frac{1}{2}\right) = 0 - \ln\frac{1}{2} = \ln 2.$

Convergent

26. $\displaystyle\int_2^\infty \frac{dv}{v^2+2v-3} = \lim_{t\to\infty}\int_2^t \frac{dv}{(v+3)(v-1)} = \lim_{t\to\infty}\int_2^t \left(\frac{-\frac{1}{4}}{v+3}+\frac{\frac{1}{4}}{v-1}\right)dv = \lim_{t\to\infty}\left[-\tfrac{1}{4}\ln|v+3|+\tfrac{1}{4}\ln|v-1|\right]_2^t$

$\displaystyle = \frac{1}{4}\lim_{t\to\infty}\left[\ln\frac{v-1}{v+3}\right]_2^t = \frac{1}{4}\lim_{t\to\infty}\left(\ln\frac{t-1}{t+3}-\ln\frac{1}{5}\right) = \frac{1}{4}(0+\ln 5) = \frac{1}{4}\ln 5.$ Convergent

27. $\int_{-\infty}^0 ze^{2z}\,dz = \lim\limits_{t\to-\infty}\int_t^0 ze^{2z}\,dz = \lim\limits_{t\to-\infty}\left[\frac{1}{2}ze^{2z}-\frac{1}{4}e^{2z}\right]_t^0$ $\left[\begin{array}{l}\text{integration by parts with}\\ u=z,\ dv=e^{2z}\,dz\end{array}\right]$

$= \lim\limits_{t\to-\infty}\left[\left(0-\frac{1}{4}\right)-\left(\frac{1}{2}te^{2t}-\frac{1}{4}e^{2t}\right)\right] = -\frac{1}{4}-0+0$ [by l'Hospital's Rule] $=-\frac{1}{4}.$ Convergent

28. $\int_2^\infty ye^{-3y}\,dy = \lim\limits_{t\to\infty}\int_2^t ye^{-3y}\,dy = \lim\limits_{t\to\infty}\left[-\frac{1}{3}ye^{-3y}-\frac{1}{9}e^{-3y}\right]_2^t$ $\left[\begin{array}{l}\text{integration by parts with}\\ u=y,\ dv=e^{-3y}\,dy\end{array}\right]$

$= \lim\limits_{t\to\infty}\left[\left(-\frac{1}{3}te^{-3t}-\frac{1}{9}e^{-3t}\right)-\left(-\frac{2}{3}e^{-6}-\frac{1}{9}e^{-6}\right)\right] = 0-0+\frac{7}{9}e^{-6}$ [by l'Hospital's Rule] $=\frac{7}{9}e^{-6}.$

Convergent

29. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx = \lim_{t\to\infty}\left[\frac{(\ln x)^2}{2}\right]_1^t$ $\left[\begin{array}{l}\text{by substitution with}\\ u=\ln x,\ du=dx/x\end{array}\right]$ $\displaystyle = \lim_{t\to\infty}\frac{(\ln t)^2}{2}=\infty.$ Divergent

30. $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty}\left[-\frac{\ln x}{x}-\frac{1}{x}\right]_1^t$ $\left[\begin{array}{l}\text{integration by parts with}\\ u=\ln x,\ dv=(1/x^2)\,dx\end{array}\right]$

$\displaystyle = \lim_{t\to\infty}\left(-\frac{\ln t}{t}-\frac{1}{t}+1\right) \overset{\text{H}}{=} \lim_{t\to\infty}\left(-\frac{1/t}{1}\right)-\lim_{t\to\infty}\frac{1}{t}+\lim_{t\to\infty}1 = 0-0+1=1.$ Convergent

31. $\displaystyle\int_{-\infty}^0 \frac{z}{z^4+4}\,dz = \lim_{t\to-\infty}\int_t^0 \frac{z}{z^4+4}\,dz = \lim_{t\to-\infty}\frac{1}{2}\left[\frac{1}{2}\tan^{-1}\left(\frac{z^2}{2}\right)\right]_t^0$ $\left[\begin{array}{l}u=z^2,\\ du=2z\,dz\end{array}\right]$

$\displaystyle = \lim_{t\to-\infty}\left[0-\frac{1}{4}\tan^{-1}\left(\frac{t^2}{2}\right)\right] = -\frac{1}{4}\left(\frac{\pi}{2}\right) = -\frac{\pi}{8}.$ Convergent

32. $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^2}\,dx = \lim_{t\to\infty}\int_e^t \frac{1}{x(\ln x)^2}\,dx = \lim_{t\to\infty}\left[-\frac{1}{\ln x}\right]_e^t$ $\left[\begin{array}{l}u=\ln x,\\ du=(1/x)\,dx\end{array}\right]$

$\displaystyle = \lim_{t\to\infty}\left(-\frac{1}{\ln t}+1\right) = 0+1=1.$ Convergent

33. $\displaystyle\int_0^\infty e^{-\sqrt{y}}\,dy = \lim_{t\to\infty}\int_0^t e^{-\sqrt{y}}\,dy = \lim_{t\to\infty}\int_0^{\sqrt{t}} e^{-x}\,(2x\,dx)$ $\left[\begin{array}{l}x=\sqrt{y},\\ dx=1/(2\sqrt{y})\,dy\end{array}\right]$

$\displaystyle = \lim_{t\to\infty}\left\{\left[-2xe^{-x}\right]_0^{\sqrt{t}}+\int_0^{\sqrt{t}} 2e^{-x}\,dx\right\}$ $\left[\begin{array}{ll}u=2x, & dv=e^{-x}\,dx\\ du=2\,dx, & v=-e^{-x}\end{array}\right]$

$\displaystyle = \lim_{t\to\infty}\left(-2\sqrt{t}\,e^{-\sqrt{t}}+\left[-2e^{-x}\right]_0^{\sqrt{t}}\right) = \lim_{t\to\infty}\left(\frac{-2\sqrt{t}}{e^{\sqrt{t}}}-\frac{2}{e^{\sqrt{t}}}+2\right) = 0-0+2=2.$

Convergent

Note: $\displaystyle\lim_{t\to\infty}\frac{\sqrt{t}}{e^{\sqrt{t}}} \overset{\text{H}}{=} \lim_{t\to\infty}\frac{2\sqrt{t}}{2\sqrt{t}\,e^{\sqrt{t}}} = \lim_{t\to\infty}\frac{1}{e^{\sqrt{t}}}=0$

Sec 7.8

34. $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x}+x\sqrt{x}}\,dx = \lim_{t\to\infty}\int_1^t \frac{dx}{\sqrt{x}\,(1+x)} = \lim_{t\to\infty}\int_1^{\sqrt{t}} \frac{1}{u(1+u^2)}\,(2u\,du)$ $\quad\left[\begin{matrix} u=\sqrt{x}, \\ du = 1/(2\sqrt{x})\,dx\end{matrix}\right]$

$\displaystyle = \lim_{t\to\infty}\int_1^{\sqrt{t}} \frac{2}{1+u^2}\,du = \lim_{t\to\infty}\left[2\tan^{-1}u\right]_1^{\sqrt{t}} = \lim_{t\to\infty} 2(\tan^{-1}\sqrt{t}-\tan^{-1}1)$

$= 2(\frac{\pi}{2}-\frac{\pi}{4}) = \frac{\pi}{2}.$ Convergent

35. $\displaystyle\int_0^1 \frac{1}{x}\,dx = \lim_{t\to 0^+}\int_t^1 \frac{1}{x}\,dx = \lim_{t\to 0^+}\Big[\ln|x|\Big]_t^1 = \lim_{t\to 0^+}(-\ln t) = \infty.$ Divergent

36. $\displaystyle\int_0^5 \frac{1}{\sqrt[3]{5-x}}\,dx = \lim_{t\to 5^-}\int_0^t (5-x)^{-1/3}\,dx = \lim_{t\to 5^-}\left[-\tfrac{3}{2}(5-x)^{2/3}\right]_0^t = \lim_{t\to 5^-}\left\{-\tfrac{3}{2}[(5-t)^{2/3}-5^{2/3}]\right\}$

$= \frac{3}{2}5^{2/3}.$ Convergent

37. $\displaystyle\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}} = \lim_{t\to -2^+}\int_t^{14} (x+2)^{-1/4}\,dx = \lim_{t\to -2^+}\left[\frac{4}{3}(x+2)^{3/4}\right]_t^{14} = \frac{4}{3}\lim_{t\to -2^+}\left[16^{3/4}-(t+2)^{3/4}\right]$

$= \frac{4}{3}(8-0) = \frac{32}{3}.$ Convergent

38. $\displaystyle\int_{-1}^2 \frac{x}{(x+1)^2}\,dx = \lim_{t\to -1^+}\int_t^2 \frac{x}{(x+1)^2}\,dx = \lim_{t\to -1^+}\int_t^2 \left[\frac{1}{x+1}-\frac{1}{(x+1)^2}\right]dx$ [partial fractions]

$\displaystyle = \lim_{t\to -1^+}\left[\ln|x+1| + \frac{1}{x+1}\right]_t^2 = \lim_{t\to -1^+}\left[\ln 3 + \frac{1}{3} - \left(\ln(t+1)+\frac{1}{t+1}\right)\right] = -\infty.$ Divergent

Note: To justify the last step, $\displaystyle\lim_{t\to -1^+}\left[\ln(t+1)+\frac{1}{t+1}\right] = \lim_{x\to 0^+}\left(\ln x + \frac{1}{x}\right)$ $\left[\begin{matrix}\text{substitute} \\ x \text{ for } t+1\end{matrix}\right]$ $\displaystyle = \lim_{x\to 0^+}\frac{x\ln x + 1}{x} = \infty$

since $\displaystyle\lim_{x\to 0^+}(x\ln x) = \lim_{x\to 0^+}\frac{\ln x}{1/x} \overset{\text{H}}{=} \lim_{x\to 0^+}\frac{1/x}{-1/x^2} = \lim_{x\to 0^+}(-x) = 0.$

39. $\displaystyle\int_{-2}^3 \frac{1}{x^4}\,dx = \int_{-2}^0 \frac{dx}{x^4} + \int_0^3 \frac{dx}{x^4}$, but $\displaystyle\int_{-2}^0 \frac{dx}{x^4} = \lim_{t\to 0^-}\left[-\frac{x^{-3}}{3}\right]_{-2}^t = \lim_{t\to 0^-}\left[-\frac{1}{3t^3}-\frac{1}{24}\right] = \infty.$ Divergent

40. $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1^-}\int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1^-}\left[\sin^{-1}x\right]_0^t = \lim_{t\to 1^-}\sin^{-1}t = \frac{\pi}{2}.$ Convergent

41. There is an infinite discontinuity at $x=1$. $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx = \int_0^1 (x-1)^{-1/3}\,dx + \int_1^9 (x-1)^{-1/3}\,dx.$

Here $\displaystyle\int_0^1 (x-1)^{-1/3}\,dx = \lim_{t\to 1^-}\int_0^t (x-1)^{-1/3}\,dx = \lim_{t\to 1^-}\left[\tfrac{3}{2}(x-1)^{2/3}\right]_0^t = \lim_{t\to 1^-}\left[\tfrac{3}{2}(t-1)^{2/3}-\tfrac{3}{2}\right] = -\tfrac{3}{2}$

and $\displaystyle\int_1^9 (x-1)^{-1/3}\,dx = \lim_{t\to 1^+}\int_t^9 (x-1)^{-1/3}\,dx = \lim_{t\to 1^+}\left[\tfrac{3}{2}(x-1)^{2/3}\right]_t^9 = \lim_{t\to 1^+}\left[6-\tfrac{3}{2}(t-1)^{2/3}\right] = 6.$ Thus,

$\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx = -\frac{3}{2}+6 = \frac{9}{2}.$ Convergent

42. There is an infinite discontinuity at $w=2$.

$\displaystyle\int_0^2 \frac{w}{w-2}\,dw = \lim_{t\to 2^-}\int_0^t\left(1+\frac{2}{w-2}\right)dw = \lim_{t\to 2^-}\Big[w+2\ln|w-2|\Big]_0^t = \lim_{t\to 2^-}(t+2\ln|t-2|-2\ln 2) = -\infty$, so

$\displaystyle\int_0^2 \frac{w}{w-2}\,dw$ diverges, and hence, $\displaystyle\int_0^5 \frac{w}{w-2}\,dw$ diverges. Divergent

43. $\int_0^{\pi/2} \tan^2\theta\,d\theta = \lim\limits_{t\to(\pi/2)^-} \int_0^t \tan^2\theta\,d\theta = \lim\limits_{t\to(\pi/2)^-} \int_0^t (\sec^2\theta - 1)\,d\theta = \lim\limits_{t\to(\pi/2)^-} \Big[\tan\theta - \theta\Big]_0^t$

$= \lim\limits_{t\to(\pi/2)^-} (\tan t - t) = \infty$ since $\tan t \to \infty$ as $t \to \frac{\pi}{2}^-$. Divergent

44. $$\int_0^4 \frac{dx}{x^2-x-2} = \int_0^4 \frac{dx}{(x-2)(x+1)} = \int_0^2 \frac{dx}{(x-2)(x+1)} + \int_2^4 \frac{dx}{(x-2)(x+1)}$$

Considering only $\displaystyle\int_0^2 \frac{dx}{(x-2)(x+1)}$ and using partial fractions, we have

$$\int_0^2 \frac{dx}{(x-2)(x+1)} = \lim_{t\to 2^-} \int_0^t \left(\frac{\frac{1}{3}}{x-2} - \frac{\frac{1}{3}}{x+1}\right) dx = \lim_{t\to 2^-} \left[\tfrac{1}{3}\ln|x-2| - \tfrac{1}{3}\ln|x+1|\right]_0^t$$

$$= \lim_{t\to 2^-} \left[\tfrac{1}{3}\ln|t-2| - \tfrac{1}{3}\ln|t+1| - \tfrac{1}{3}\ln 2 + 0\right] = -\infty \text{ since } \ln|t-2| \to -\infty \text{ as } t \to 2^-.$$

Thus, $\displaystyle\int_0^2 \frac{dx}{x^2-x-2}$ is divergent, and hence, $\displaystyle\int_0^4 \frac{dx}{x^2-x-2}$ is divergent as well.

45. $\int_0^1 r\ln r\,dr = \lim\limits_{t\to 0^+} \int_t^1 r\ln r\,dr = \lim\limits_{t\to 0^+} \left[\frac{1}{2}r^2\ln r - \frac{1}{4}r^2\right]_t^1$ $\quad\left[\begin{matrix} u = \ln r, & dv = r\,dr \\ du = (1/r)\,dr, & v = \frac{1}{2}r^2 \end{matrix}\right]$

$= \lim\limits_{t\to 0^+} \left[\left(0 - \frac{1}{4}\right) - \left(\frac{1}{2}t^2\ln t - \frac{1}{4}t^2\right)\right] = -\frac{1}{4} - 0 = -\frac{1}{4}$

since $\lim\limits_{t\to 0^+} t^2\ln t = \lim\limits_{t\to 0^+} \dfrac{\ln t}{1/t^2} \overset{\text{H}}{=} \lim\limits_{t\to 0^+} \dfrac{1/t}{-2/t^3} = \lim\limits_{t\to 0^+} \left(-\frac{1}{2}t^2\right) = 0.$ Convergent

46. $\displaystyle\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{\sin\theta}}\,d\theta = \lim_{t\to 0^+} \int_t^{\pi/2} \frac{\cos\theta}{\sqrt{\sin\theta}}\,d\theta = \lim_{t\to 0^+} \Big[2\sqrt{\sin\theta}\Big]_t^{\pi/2}$ $\quad\left[\begin{matrix} u = \sin\theta, \\ du = \cos\theta\,d\theta \end{matrix}\right]$

$= \lim\limits_{t\to 0^+} (2 - 2\sqrt{\sin t}) = 2 - 0 = 2.$ Convergent

47. $\displaystyle\int_{-1}^0 \frac{e^{1/x}}{x^3}\,dx = \lim_{t\to 0^-} \int_{-1}^t \frac{1}{x}e^{1/x}\cdot\frac{1}{x^2}\,dx = \lim_{t\to 0^-} \int_{-1}^{1/t} ue^u\,(-du)$ $\quad\left[\begin{matrix} u = 1/x, \\ du = -dx/x^2 \end{matrix}\right]$

$= \lim\limits_{t\to 0^-} \Big[(u-1)e^u\Big]_{1/t}^{-1}$ $\left[\begin{matrix}\text{use parts} \\ \text{or Formula 96}\end{matrix}\right]$ $= \lim\limits_{t\to 0^-} \left[-2e^{-1} - \left(\dfrac{1}{t} - 1\right)e^{1/t}\right]$

$= -\dfrac{2}{e} - \lim\limits_{s\to -\infty} (s-1)e^s \quad [s = 1/t] \quad = -\dfrac{2}{e} - \lim\limits_{s\to -\infty} \dfrac{s-1}{e^{-s}} \overset{\text{H}}{=} -\dfrac{2}{e} - \lim\limits_{s\to -\infty} \dfrac{1}{-e^{-s}}$

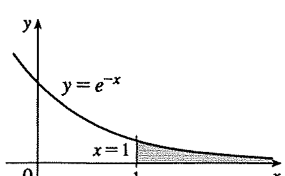
$= -\dfrac{2}{e} - 0 = -\dfrac{2}{e}.$ Convergent

48. $\displaystyle\int_0^1 \frac{e^{1/x}}{x^3}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{1}{x}e^{1/x}\cdot\frac{1}{x^2}\,dx = \lim_{t\to 0^+} \int_{1/t}^1 ue^u\,(-du)$ $\quad\left[\begin{matrix} u = 1/x, \\ du = -dx/x^2 \end{matrix}\right]$

$= \lim\limits_{t\to 0^+} \Big[(u-1)e^u\Big]_1^{1/t}$ $\left[\begin{matrix}\text{use parts} \\ \text{or Formula 96}\end{matrix}\right]$ $= \lim\limits_{t\to 0^+} \left[\left(\dfrac{1}{t} - 1\right)e^{1/t} - 0\right]$

$= \lim\limits_{s\to\infty} (s-1)e^s \quad [s = 1/t] \quad = \infty.$ Divergent

49.



Area $= \int_1^\infty e^{-x}\,dx = \lim\limits_{t\to\infty} \int_1^t e^{-x}\,dx = \lim\limits_{t\to\infty} \Big[-e^{-x}\Big]_1^t$

$= \lim\limits_{t\to\infty} (-e^{-t} + e^{-1}) = 0 + e^{-1} = 1/e$